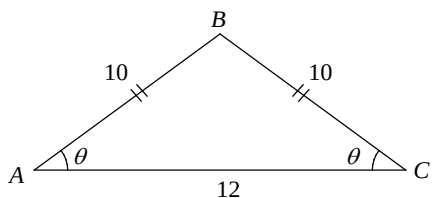


Exercises - Trigonometric Ratios of Acute Angles

Questions 1- 3 refer to the following information.

In the triangle shown below $AB = BC = 10$ and $AC = 12$.



1

What is the value of $\cos \theta$?

- A) 0.4
- B) 0.6
- C) 0.8
- D) 1.2

2

What is the value of $\sin \theta$?

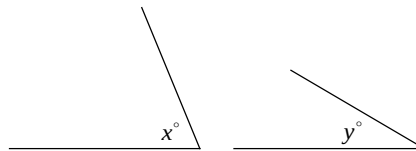
- A) 0.4
- B) 0.6
- C) 0.8
- D) 1.2

3

What is the value of $\tan \theta$?

- A) $\frac{3}{4}$
- B) $\frac{4}{3}$
- C) $\frac{5}{4}$
- D) $\frac{5}{3}$

4

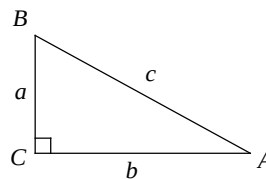


Note: Figures not drawn to scale.

In the figures above $y < x < 90$ and $\cos x^\circ = \sin y^\circ$. If $x = 3a - 14$ and $y = 50 - a$, what is the value of a ?

- A) 16
- B) 21
- C) 24
- D) 27

5

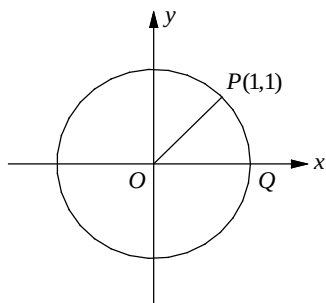


Given the right triangle ABC above, which of the following is equal to $\frac{a}{c}$?

- I. $\sin A$
 - II. $\cos B$
 - III. $\tan A$
- A) I only
 - B) II only
 - C) I and II only
 - D) II and III only

Exercises - The Radian Measure of an Angle

1



In the xy -plane above, O is the center of the circle, and the measure of $\angle POQ$ is $k\pi$ radians. What is the value of k ?

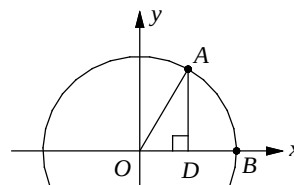
- A) $\frac{1}{6}$
- B) $\frac{1}{4}$
- C) $\frac{1}{3}$
- D) $\frac{1}{2}$

2

Which of the following is equal to $\cos(\frac{\pi}{8})$?

- A) $\cos(\frac{3\pi}{8})$
- B) $\cos(\frac{7\pi}{8})$
- C) $\sin(\frac{3\pi}{8})$
- D) $\sin(\frac{7\pi}{8})$

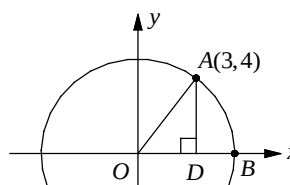
3



In the xy -plane above, O is the center of the circle and the measure of $\angle AOD$ is $\frac{\pi}{3}$. If the radius of circle O is 6 what is the length of AD ?

- A) 3
- B) $3\sqrt{2}$
- C) 4.5
- D) $3\sqrt{3}$

4

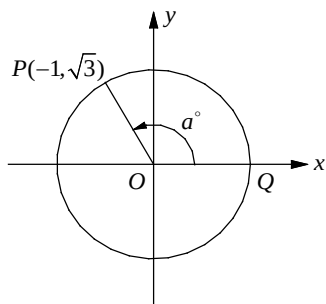


In the figure above, what is the value of $\cos \angle AOD$?

- A) $\frac{3}{5}$
- B) $\frac{3}{4}$
- C) $\frac{4}{5}$
- D) $\frac{4}{3}$

Exercises - The Trigonometric Functions and the Unit Circle

Questions 1 and 2 refer to the following information.



In the xy -plane above, O is the center of the circle, and the measure of $\angle POQ$ is a° .

1

What is the cosine of a° ?

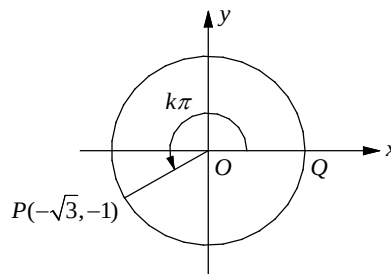
- A) $-\frac{1}{2}$
- B) $\sqrt{3}$
- C) $-\frac{1}{\sqrt{3}}$
- D) $\frac{\sqrt{3}}{2}$

2

What is the cosine of $(a+180)^\circ$?

- A) $-\sqrt{3}$
- B) $-\frac{\sqrt{3}}{2}$
- C) $\frac{1}{2}$
- D) $\frac{1}{\sqrt{3}}$

Questions 3 and 4 refer to the following information.



In the xy -plane above, O is the center of the circle, and the measure of the angle shown is $k\pi$ radians.

3

What is the value of k ?

- A) $\frac{5}{6}$
- B) $\frac{7}{6}$
- C) $\frac{4}{3}$
- D) $\frac{5}{3}$

4

What is the value of $\tan(k\pi)$?

- A) $-\sqrt{3}$
- B) -1
- C) $-\frac{1}{\sqrt{3}}$
- D) $\frac{1}{\sqrt{3}}$

Answer Key

Section 15-1

1. B 2. C 3. B 4. D 5. C

Section 15-2

1. B 2. C 3. D 4. A

Section 15-3

1. A 2. C 3. B 4. D

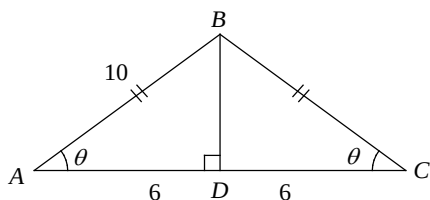
Chapter 15 Practice Test

1. D 2. C 3. C 4. B 5. D
6. D 7. $\frac{5}{13}$ 8. 9 9. 10.5

Answers and Explanations

Section 15-1

1. B



Draw a perpendicular segment from B to the opposite side AC . Let the perpendicular segment intersect side AC at D . Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

Therefore, $AD = \frac{1}{2}AC = \frac{1}{2}(12) = 6$.

In right $\triangle ABD$,

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{6}{10} = 0.6.$$

2. C

$$AB^2 = BD^2 + AD^2 \quad \text{Pythagorean Theorem}$$

$$10^2 = BD^2 + 6^2$$

$$100 = BD^2 + 36$$

$$64 = BD^2$$

$$8 = BD$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{8}{10} = 0.8$$

3. B

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BD}{AD} = \frac{8}{6} = \frac{4}{3}$$

4. D

If x and y are acute angles and $\cos x^\circ = \sin y^\circ$, $x + y = 90$ by the complementary angle theorem.

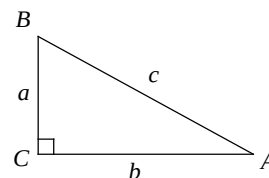
$$(3a - 14) + (50 - a) = 90 \quad x = 3a - 14, \quad y = 50 - a$$

$$2a + 36 = 90 \quad \text{Simplify.}$$

$$2a = 54$$

$$a = 27$$

5. C



$$\text{I. } \sin A = \frac{\text{opposite of } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

Roman numeral I is true.

$$\text{II. } \cos B = \frac{\text{adjacent of } \angle B}{\text{hypotenuse}} = \frac{a}{c}$$

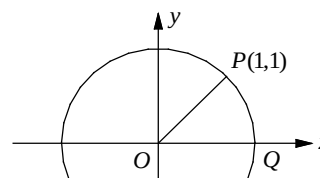
Roman numeral II is true.

$$\text{III. } \tan A = \frac{\text{opposite of } \angle A}{\text{adjacent of } \angle A} = \frac{a}{b}$$

Roman numeral III is false.

Section 15-2

1. B



The graph shows $P(x, y) = P(1, 1)$. Thus, $x = 1$ and $y = 1$. Use the distance formula to find the length of radius OA .

$$OA = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \quad \text{or} \quad \sin \theta = \frac{\sqrt{2}}{2}$$

Therefore, the measure of $\angle POQ$ is 45° ,
which is equal to $45\left(\frac{\pi}{180}\right) = \frac{\pi}{4}$ radians.

Thus, $k = \frac{1}{4}$.

2. C

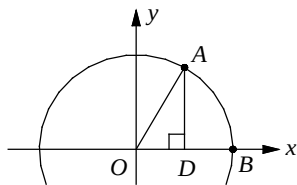
Use the complementary angle theorem.

$$\cos(\theta) = \sin(90^\circ - \theta), \text{ or } \cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\text{Therefore, } \cos\left(\frac{\pi}{8}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \sin\left(\frac{3\pi}{8}\right).$$

All the other answer choices have values
different from $\cos\left(\frac{\pi}{8}\right)$.

3. D

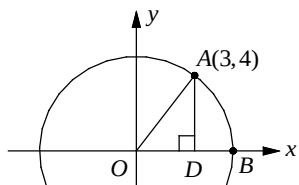


$$\text{In } \triangle OAD, \sin \frac{\pi}{3} = \sin 60^\circ = \frac{AD}{OA} = \frac{AD}{6}.$$

$$\text{Since } \sin 60^\circ = \frac{\sqrt{3}}{2}, \text{ you get } \frac{AD}{6} = \frac{\sqrt{3}}{2}.$$

$$\text{Therefore, } 2AD = 6\sqrt{3} \text{ and } AD = 3\sqrt{3}.$$

4. A



Use the distance formula to find the length of OA .

$$OA = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\cos \angle AOD = \frac{OD}{OA} = \frac{3}{5}$$

Section 15-3

1. A

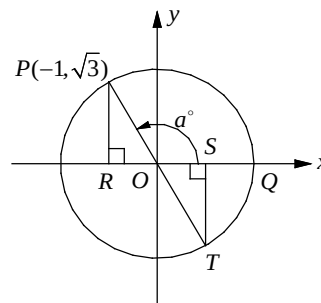
Draw segment PR , which is perpendicular to
the x -axis. In right triangle POR , $x = -1$

and $y = \sqrt{3}$. To find the length of OP , use the
Pythagorean theorem.

$$OP^2 = PR^2 + OR^2 = (\sqrt{3})^2 + (-1)^2 = 4$$

Which gives $OP = 2$.

$$\cos a^\circ = \frac{x}{OP} = \frac{-1}{2}$$

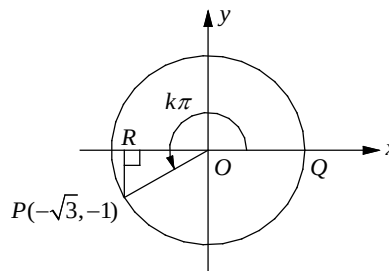


2. C

Since the terminal side of $(a+180)^\circ$ is OT , the
value of $\cos(a+180)^\circ$ is equal to $\frac{OS}{OT}$.

$$\frac{OS}{OT} = \frac{1}{2}$$

3. B



Draw segment PR , which is perpendicular to
the x -axis. In right triangle POR , $x = -\sqrt{3}$
and $y = -1$. To find the length of OP , use the
Pythagorean theorem.

$$OP^2 = PR^2 + OR^2 = (-1)^2 + (\sqrt{3})^2 = 4$$

Which gives $OP = 2$.

$$\text{Since } \sin \angle POR = \frac{y}{OP} = \frac{-1}{2}, \text{ the measure of}$$

$\angle POR$ is equal to 30° , or $\frac{\pi}{6}$ radian.

$$k\pi = \pi + \frac{\pi}{6} = \frac{7}{6}\pi$$

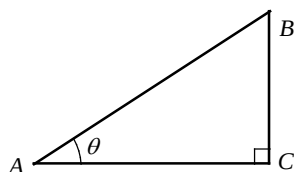
$$\text{Therefore, } k = \frac{7}{6}$$

4. D

$$\tan(k\pi) = \tan\left(\frac{7}{6}\pi\right) = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Chapter 15 practice Test

1. D



Note: Figure not drawn to scale.

$$\text{In } \triangle ABC, \tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC}.$$

If $\tan \theta = \frac{3}{4}$, then $BC = 3$ and $AC = 4$.

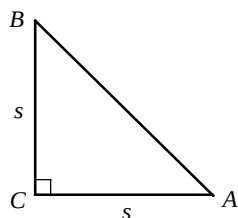
By the Pythagorean theorem,

$$AB^2 = AC^2 + BC^2 = 4^2 + 3^2 = 25, \text{ thus}$$

$$AB = \sqrt{25} = 5.$$

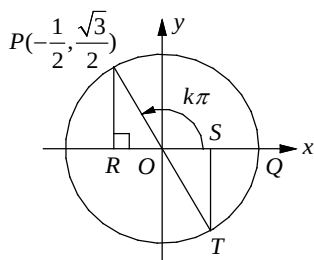
$$\sin \theta = \frac{BC}{AB} = \frac{3}{5}$$

2. C



$$\begin{aligned} \tan \angle A &= \frac{\text{opposite side of } \angle A}{\text{adjacent side of } \angle A} = \frac{s}{s} = 1 \\ &= \frac{s}{s} = 1 \end{aligned}$$

3. C



Draw segment PR , which is perpendicular to the x -axis. In right triangle POR , $x = -\frac{1}{2}$

and $y = \frac{\sqrt{3}}{2}$. To find the length of OP , use the Pythagorean theorem.

$$OP^2 = PR^2 + OR^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$$

Which gives $OP = 1$. Thus, triangle OPR is 30° - 60° - 90° triangle and the measure of $\angle POR$ is 60° , which is $\frac{\pi}{3}$ radian. Therefore, the measure

of $\angle POQ$ is $\pi - \frac{\pi}{3}$, or $\frac{2\pi}{3}$ radian. If $\angle POQ$ is

$k\pi$ radians then k is equal to $\frac{2}{3}$.

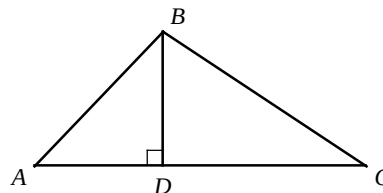
4. B

Since the terminal side of $(k+1)\pi$ is OT , the

value of $\cos(k+1)\pi$ is equal to $\frac{OS}{OT}$.

$$\frac{OS}{OT} = \frac{1}{2}$$

5. D



$$\text{Area of triangle } ABC = \frac{1}{2}(AC)(BD)$$

Check each answer choice.

$$\text{A) } \frac{1}{2}(AB \cos \angle A + BC \cos \angle C)(AB \cos \angle ABD)$$

$$= \frac{1}{2}\left(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC}\right)\left(AB \cdot \frac{BD}{AB}\right)$$

$$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$$

$$\text{B) } \frac{1}{2}(AB \cos \angle A + BC \cos \angle C)(BC \sin \angle C)$$

$$= \frac{1}{2}\left(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC}\right)\left(BC \cdot \frac{BD}{BC}\right)$$

$$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$$

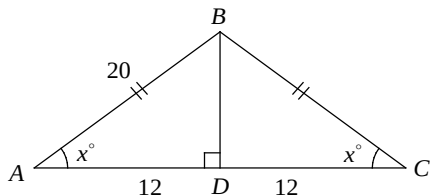
$$\begin{aligned}
 \text{C) } & \frac{1}{2}(AB \sin \angle ABD + BC \sin \angle CBD)(AB \sin \angle A) \\
 &= \frac{1}{2}\left(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC}\right)\left(AB \cdot \frac{BD}{AB}\right) \\
 &= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)
 \end{aligned}$$

$$\begin{aligned}
 \text{D) } & \frac{1}{2}(AB \sin \angle ABD + BC \sin \angle CBD)(BC \cos \angle C) \\
 &= \frac{1}{2}\left(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC}\right)\left(BC \cdot \frac{CD}{BC}\right) \\
 &= \frac{1}{2}(AD + CD)(CD) = \frac{1}{2}(AC)(CD)
 \end{aligned}$$

Which does not represent the area of triangle ABC .

Choice D is correct.

6. D



Draw segment BD , which is perpendicular to side AC . Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

$$\text{Therefore, } AD = \frac{1}{2}AC = \frac{1}{2}(24) = 12.$$

By the Pythagorean theorem, $AB^2 = BD^2 + AD^2$

$$\text{Thus, } 20^2 = BD^2 + 12^2.$$

$$BD^2 = 20^2 - 12^2 = 256$$

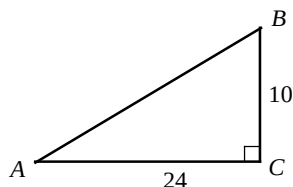
$$BD = \sqrt{256} = 16$$

In right $\triangle ABD$,

$$\sin x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{16}{20} = \frac{4}{5}.$$

7. $\frac{5}{13}$

Sketch triangle ABC .



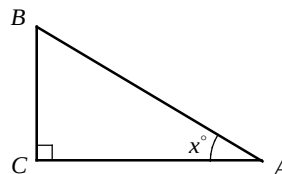
$$AB^2 = BC^2 + AC^2$$

$$AB^2 = 10^2 + 24^2 = 676$$

$$AB = \sqrt{676} = 26$$

$$\sin A = \frac{10}{26} = \frac{5}{13}$$

8. 9



$$\cos x^\circ = \frac{AC}{AB} = \frac{3}{5}$$

Let $AC = 3x$ and $AB = 5x$.

$$AB^2 = BC^2 + AC^2 \quad \text{Pythagorean Theorem}$$

$$(5x)^2 = 12^2 + (3x)^2 \quad BC = 12$$

$$25x^2 = 144 + 9x^2$$

$$16x^2 = 144$$

$$x^2 = 9$$

$$x = \sqrt{9} = 3$$

Therefore, $AC = 3x = 3(3) = 9$

9. 10.5

According to the complementary angle theorem, $\sin \theta = \cos(90 - \theta)$.

$$\text{If } \sin(5x - 10)^\circ = \cos(3x + 16)^\circ,$$

$$3x + 16 = 90 - (5x - 10).$$

$$3x + 16 = 90 - 5x + 10$$

$$3x + 16 = 100 - 5x$$

$$8x = 84$$

$$x = 10.5$$