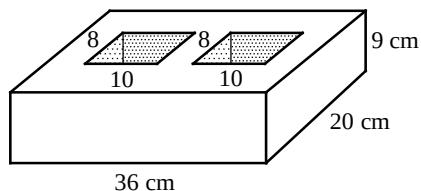


Exercises – Prisms

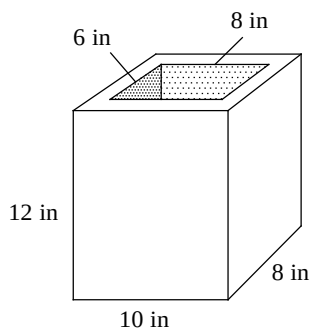
1



The figure above shows a cement block of $36\text{ cm} \times 20\text{ cm} \times 9\text{ cm}$ with two $10\text{ cm} \times 8\text{ cm}$ openings. What is the weight of the cement block to the nearest gram? (The density of cement is $1.7\text{ gram} / \text{cm}^3$)

- A) 5,040
- B) 6,048
- C) 7,560
- D) 8,568

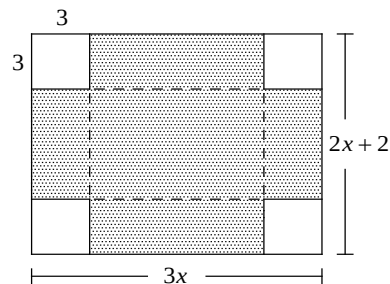
2



The figure above shows an aluminum block of $10\text{ in} \times 8\text{ in} \times 12\text{ in}$ with an $8\text{ in} \times 6\text{ in} \times 12\text{ in}$ opening. What is the weight of the aluminum block to the nearest pound? (The density of aluminum is 0.098 lb/in^3)

- A) 32
- B) 38
- C) 42
- D) 48

3



A manufacturing company produces cardboard boxes by cutting out square corners 3 inches (in) by 3 in. from rectangular pieces of cardboard $3x$ in. by $2x + 2$ in. The cardboard is then folded along the dashed lines to form a box without a top. If the volume of the box is 162 in^3 , what is the dimension of the original cardboard before cutting out its square corners?

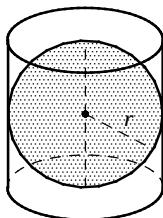
- A) $12\text{ in} \times 9\text{ in}$
- B) $14\text{ in} \times 10\text{ in}$
- C) $15\text{ in} \times 12\text{ in}$
- D) $16\text{ in} \times 14\text{ in}$

4

An aquarium tank in the shape of a rectangular prism is 20 inches (in) long by 16 in wide by 12 in high. If 2,400 cubic inches of water is added into the empty tank, how far is the surface of the water from the top of the tank?

Exercises - Cylinders and Spheres

1

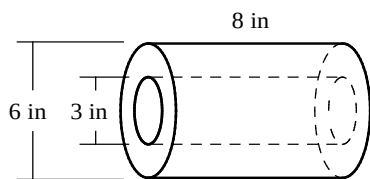


In the figure above, a sphere is inscribed in a cylinder, so that the diameter of the sphere is the same as the diameter of the cylinder and the height of the cylinder. What is the value

of $\frac{\text{Volume of the sphere}}{\text{Volume of the cylinder}}$?

- A) $\frac{1}{2}$
- B) $\frac{2}{3}$
- C) $\frac{7}{10}$
- D) $\frac{3}{4}$

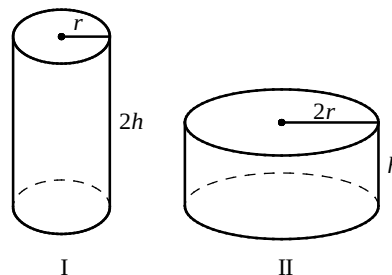
2



The figure above shows the mechanical part in the shape of a steel cylinder 8 inches high and 6 inches long in diameter. A hole with a diameter of 3 inches is drilled through the mechanical part. The density of steel is 490 lb/ft^3 . What is the mass of the mechanical part, to the nearest pound? (1 foot = 12 inch)

- A) 36
- B) 42
- C) 48
- D) 52

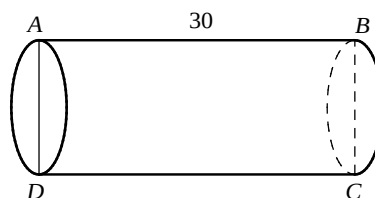
3



The figure above shows two cylinders. The height of cylinder I is twice the height of cylinder II and the radius of cylinder II is twice the radius of cylinder I. If the volume of cylinder I is $45\pi \text{ in}^3$, what is the volume of cylinder II in cubic inches?

- A) 22.5π
- B) 45π
- C) 67.5π
- D) 90π

4

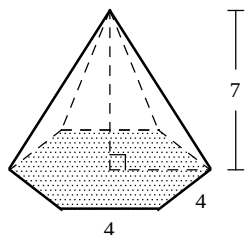


In the cylindrical tube shown above, the height of the tube is 30 and the circumference of the circular base is 32. If the tube is cut along $\overline{AB}$ and laid out flat to make a rectangle, what is the length of $\overline{AC}$ to the nearest whole number?

- A) 24
- B) 30
- C) 34
- D) 38

Exercises - Pyramids and Cones

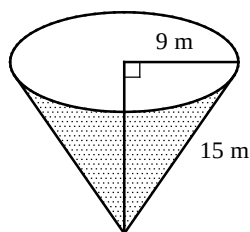
1



The figure above shows a pyramid with regular hexagonal base. The length of each side of the hexagonal face is 4 units and the height of the pyramid is 7 units. What is the volume of the pyramid?

- A) $35\sqrt{3}$
- B) $56\sqrt{3}$
- C) $84\sqrt{3}$
- D) $168\sqrt{3}$

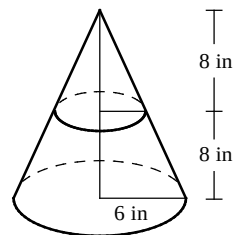
2



Water is pouring into a conical reservoir at the rate of 2.4 m^3 per minute. If the radius of the base of the conical reservoir is 9 meters (m) and the length of the lateral edge is 15 m, to the nearest minute, how long will it take to fill up the empty reservoir?

- A) 212
- B) 318
- C) 424
- D) 530

3



A plane parallel to the base of a cone divides the cone into two pieces, and removes the top part. The radius of the cone is 6 inches (in), the height of the cone is 16 in, and the distance from the base to the parallel plane is 8 in. What is the volume of the remaining bottom part, in cubic inches?

- A) 56π
- B) 84π
- C) 126π
- D) 168π

4

If the circumference of the base part of cone is 10 centimeters (cm) and the height of the cone is 8 cm, what is the volume of the cone, to the nearest cubic centimeter?

- A) 18
- B) 21
- C) 24
- D) 32

Answer Key

Section 20-1

1. D 2. B 3. C 4. 4.5

Section 20-2

1. B 2. C 3. D 4. C

Section 20-3

1. B 2. C 3. D 4. B

Chapter 20 Practice Test

1. D 2. A 3. B 4. D 5. C
6. C 7. 27 8. 85 9. $\frac{16}{3}$ or 5.33

Answers and Explanations

Section 20-1

1. D

Volume of the block is $36 \times 20 \times 9 - 2(10 \times 8 \times 9)$,
or $5,040 \text{ cm}^3$.

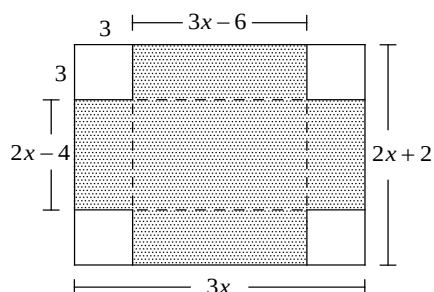
Weight of the cement block = density $\times$ volume
 $= 1.7 \text{ gram / cm}^3 \times 5,040 \text{ cm}^3 = 8,568 \text{ gram}$.

2. B

Volume of the aluminum block is
 $10 \times 8 \times 12 - 8 \times 6 \times 12$, or 384 in^3 .

Weight of the aluminum block = density $\times$ volume
 $= 0.098 \text{ lb / in}^3 \times 384 \text{ in}^3 = 37.632 \text{ lb}$.

3. C



The dimension of the box is $(3x-6) \times (2x-4) \times 3$.
Since the volume of the box is given as 162 in^3 ,
set up the following equation:

$$(3x-6) \times (2x-4) \times 3 = 162$$

$$(3x-6) \times (2x-4) = 54$$

$$6x^2 - 24x + 24 = 54$$

$$6x^2 - 24x - 30 = 0$$

$$6(x^2 - 4x - 5) = 0$$

$$6(x-5)(x+1) = 0$$

$$x = 5 \text{ or } x = -1$$

The length of the side is positive, so $x = 5$.
Therefore, the dimension of the original cardboard
is $(3 \times 5) \text{ in} \times (2 \times 5 + 2) \text{ in}$, or $15 \text{ in} \times 12 \text{ in}$.

4. 4.5

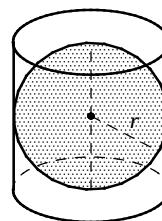
Let h be the height of water when 2,400 cubic
inches of water is added into the empty tank.
Then $20 \times 16 \times h = 2,400$. Solving for h yields

$$h = \frac{2,400}{20 \times 16} = 7.5 \text{ in.}$$

Since the aquarium tank is 12 inches high, the
surface of water will be $12 - 7.5$, or 4.5 inches,
from the top of the tank.

Section 20-2

1. B



$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of the cylinder} = \pi r^2 \cdot \text{height}$$

$$= \pi r^2 \cdot 2r$$

$$= 2\pi r^3$$

$$\frac{\text{Volume of the sphere}}{\text{Volume of the cylinder}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

2. C

$$\text{Volume of the mechanical part} \\ = \pi(3)^2 \cdot 8 - \pi(1.5)^2 \cdot 8 = 54\pi \text{ in}^3$$

$$\text{Since } 1 \text{ ft} = 12 \text{ in}, 1 \text{ ft}^3 = (12 \text{ in})^3 = 1,728 \text{ in}^3.$$

$$\text{Thus } 1 \text{ in}^3 = \frac{1}{1,728} \text{ ft}^3.$$

$$\begin{aligned}\text{Mass of the mechanical part} &= \text{density} \times \text{volume} \\ &= 490 \text{ lb / ft}^3 \times 54\pi \text{ in}^3 \cdot \frac{1 \text{ ft}^3}{1,728 \text{ in}^3} \approx 48.1 \text{ lb}.\end{aligned}$$

3. D

$$\text{Volume of cylinder I} = \pi(r)^2 \cdot 2h = 2\pi r^2 h$$

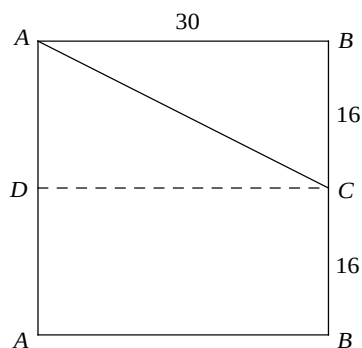
$$\text{Volume of cylinder II} = \pi(2r)^2 \cdot h = 4\pi r^2 h$$

$$\frac{\text{volume of cylinder I}}{\text{volume of cylinder II}} = \frac{2\pi r^2 h}{4\pi r^2 h} = \frac{1}{2}$$

Thus if the volume of cylinder I is $45\pi \text{ in}^3$, the volume of cylinder II is $90\pi \text{ in}^3$.

4. C

The figure below shows the rectangle, which was laid flat from a cut along $\overline{AB}$ on the cylinder.



$$AC^2 = AB^2 + BC^2 \quad \text{Pythagorean Theorem}$$

$$AC^2 = 30^2 + 16^2 \quad \text{Substitution}$$

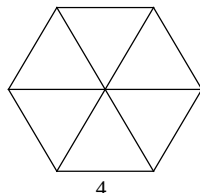
$$= 900 + 256$$

$$= 1,156$$

$$AC = \sqrt{1,156} = 34$$

Section 20-3

1. B



The regular hexagon consists of 6 equilateral triangles. So the area of the regular hexagon is the sum of the areas of 6 equilateral triangles. Since the area of the equilateral triangle with side

length of a is $\frac{\sqrt{3}}{4}a^2$, the area of the equilateral

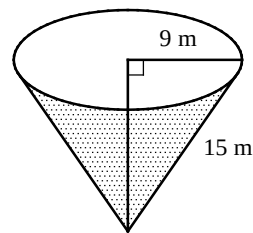
triangle with side length of 4 is $\frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$.

The area of the hexagon is $6 \times 4\sqrt{3} = 24\sqrt{3}$.

Volume of the pyramid

$$= \frac{1}{3}Bh = \frac{1}{3}(24\sqrt{3})(7) = 56\sqrt{3}$$

2. C



Let h = the height of the cone.

$$9^2 + h^2 = 15^2$$

Pythagorean Theorem

$$h^2 = 15^2 - 9^2 = 144$$

$$h = \sqrt{144} = 12$$

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(9)^2(12) = 324\pi \text{ m}^3$$

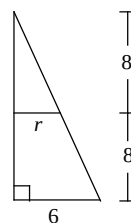
Number of minutes it takes to fill up the reservoir

= total volume $\div$ rate of filling

$$= 324\pi \text{ m}^3 \div 2.4 \text{ m}^3 / \text{min}$$

$$\approx 424.1 \text{ min}$$

3. D



Let r = the radius of the smaller cone.

Since the bases are parallel, the proportion

$$\frac{8}{r} = \frac{16}{6} \quad \text{can be used to find the radius of the}$$

$$\text{smaller cone. } \frac{8}{r} = \frac{16}{6} \Rightarrow r = 3$$

Volume of the remaining bottom part

= volume of the cone – volume of the top part

$$= \frac{1}{3}\pi(6)^2(16) - \frac{1}{3}\pi(3)^2(8) = 168\pi$$

4. B

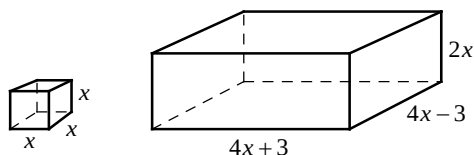
$$C = 2\pi r = 10 \Rightarrow r = \frac{10}{2\pi} = \frac{5}{\pi}$$

Volume of the cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{5}{\pi}\right)^2 (8) = \frac{200}{3\pi} \approx 21.2$$

Chapter 20 Practice Test

1. D



Volume of the rectangular prism

$$= (4x+3)(4x-3)(2x) = (16x^2-9)(2x)$$

Volume of the cube = x^3

Since the volume of the rectangular prism is 30 times the volume of the cube, the equation $(16x^2-9)(2x) = 30x^3$ can be used to find the value of x .

$$(16x^2-9)(2x) - 30x^3 = 0 \quad \text{Make one side 0.}$$

$$2x[(16x^2-9) - 15x^2] = 0 \quad \text{GCF is } 2x.$$

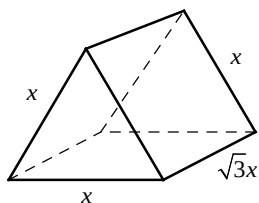
$$2x(x^2-9) = 0 \quad \text{Simplify.}$$

$$2x(x+3)(x-3) = 0 \quad \text{Factor.}$$

$$x = 0, x = -3, \text{ and } x = 3$$

Since the dimension has to be positive, $x = 3$ is the correct answer.

2. A



Area of the equilateral triangle with side length of x is $\frac{\sqrt{3}}{4}x^2$.

Volume of the triangular prism

$$= B \cdot h = \frac{\sqrt{3}}{4}x^2 \cdot \sqrt{3}x = \frac{3}{4}x^3$$

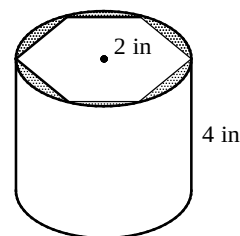
Since the volume of the prism is given as $\frac{81}{4}$,

the equation $\frac{3}{4}x^3 = \frac{81}{4}$ can be used to find the value of x .

$$\frac{3}{4}x^3 = \frac{81}{4} \Rightarrow 3x^3 = 81 \Rightarrow x^3 = 27$$

$$\Rightarrow x = \sqrt[3]{27} = 3$$

3. B



Area of the equilateral triangle with side length of 2 = $\frac{\sqrt{3}}{4}(2)^2 = \sqrt{3}$.

Area of the regular hexagon = $6\sqrt{3}$.

Volume of the hexagonal prism = $6\sqrt{3} \cdot 4 = 24\sqrt{3}$

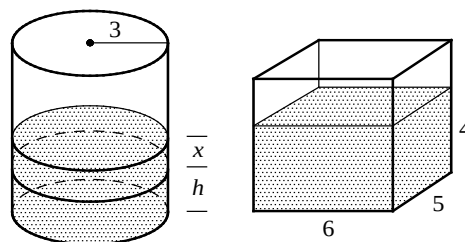
Volume of the cylinder = $\pi(2)^2 \cdot 4 = 16\pi$

The volume of the waste generated by creating the hexagonal prism from the cylinder can be found by subtracting the volume of the hexagonal prism from the volume of the cylinder.

$$16\pi - 24\sqrt{3} \approx 8.69$$

The volume of the waste is about 9 cubic inches.

4. D



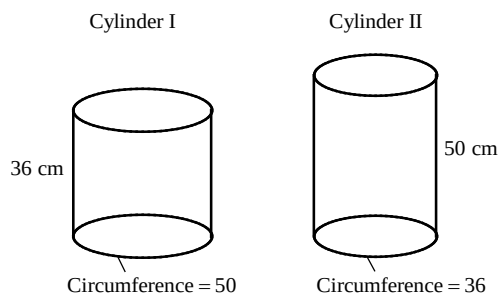
The volume of the cylinder with a radius of 3 and a height of x is $\pi(3)^2x$, or $9\pi x$.

Volume of the water in the rectangular container is $6 \times 5 \times 4$, or 120.

To solve for x , let $9\pi x = 120$.

$$x = \frac{120}{9\pi} \approx 4.24$$

5. C



Let r_1 = the radius of cylinder I and
let r_2 = the radius of cylinder II.

$$2\pi r_1 = 50 \Rightarrow r_1 = \frac{50}{2\pi} = \frac{25}{\pi}$$

$$2\pi r_2 = 36 \Rightarrow r_2 = \frac{36}{2\pi} = \frac{18}{\pi}$$

Volume of cylinder I

$$= \pi(r_1)^2 h = \pi\left(\frac{25}{\pi}\right)^2 (36) = \frac{22,500}{\pi}$$

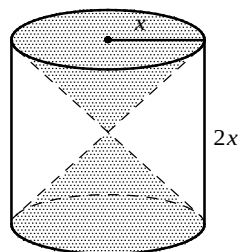
Volume of cylinder II

$$= \pi(r_2)^2 h = \pi\left(\frac{18}{\pi}\right)^2 (50) = \frac{16,200}{\pi}$$

The difference of volume between the two cylinders is $\frac{22,500}{\pi} - \frac{16,200}{\pi} \approx 2,005.3$.

Choice C is correct.

6. C



Volume of the space inside the cylinder but outside the double cone =
volume of the cylinder – volume of the two cones.

$$\pi(x)^2(2x) - 2\left[\frac{1}{3}\pi(x)^2(x)\right]$$

$$= 2\pi x^3 - \frac{2}{3}\pi x^3 = \frac{4}{3}\pi x^3$$

7. 27

Surface area of the cube = $6s^2$

Since the surface area of the cube is given as 54 cm^2 , $6s^2 = 54$.

$6s^2 = 54$ is simplified to $s^2 = 9$. Solving for s gives $s = 3$.

$$\text{Volume of the cube} = s^3 = (3)^3 = 27$$

8. 85

$$\text{Volume of the cone} = \frac{1}{3}\pi(3)^2(10) = 30\pi$$

Since the cone is 90 percent filled with shaved ice, the volume of the shaved ice is $30\pi \times 0.9$, or 27π cubic centimeters.

$$27\pi \text{ cm}^3 \approx 84.8 \text{ cm}^3$$

Therefore, to the nearest cubic centimeter, the volume of the shaved is 85 cm^3 .

9. $\frac{16}{3}$ or 5.33

Let h = the height of the square pyramid.

Volume of the square pyramid

$$= \frac{1}{3}Bh = \frac{1}{3}(6)^2 h = 12h$$

$$\text{Volume of the cube} = s^3 = (4)^3 = 64$$

Since the square pyramid and the cube have equal volumes, $12h = 64$.

$$\text{Solving for } h \text{ gives } h = \frac{64}{12} = \frac{16}{3}, \text{ or } 5.33.$$