

Exercises - Ratios and Rates

1

The ratio of $1\frac{3}{4}$ to $2\frac{1}{2}$ is equal to the ratio of 14 to what number?

- A) 18
- B) 20
- C) 22
- D) 24

2

The sum of two numbers is 14 and the ratio of the two numbers is -3 . What is the product of the two numbers?

- A) -105
- B) -119
- C) -133
- D) -147

3

If $2(x - y) = 3y$, what is the ratio $\frac{x}{y}$?

- A) $\frac{2}{5}$
- B) $\frac{4}{3}$
- C) $\frac{5}{2}$
- D) $\frac{8}{3}$

4

The ratio of length to width of a rectangular garden is $6:7$. If the perimeter of the rectangle is 78 meters, what is the area of the garden in square meters?

- A) 274
- B) 326
- C) 352
- D) 378

5

A car travels 218.5 miles on 9.5 gallons of gas. What is the car's gas mileage?

6

At a grocery store, 20 fl oz of brand A vitamin water is sold for \$0.95. What is the unit price of the vitamin water per ounce, to the nearest cents?

7

The density of aluminum is 2.7 grams per cm^3 . How many grams does 12 cm^3 of aluminum weigh?

Exercises - Proportions

1

On a map, 1 inch represents 5 miles. If a certain state is represented on a map by a rectangle 10 inches by 7.2 inches, what is the area of the state in square miles?

- A) 360 mi^2
- B) 720 mi^2
- C) $1,080 \text{ mi}^2$
- D) $1,800 \text{ mi}^2$

2

Together there are 754 students and teachers in the meeting. If the ratio of students to teachers is $27 : 2$, how many teachers are there?

- A) 46
- B) 52
- C) 58
- D) 64

3

Concrete is made by mixing cement, sand, and gravel in the ratio $5 : 9 : 13$. How much cement is needed to make 324 ft^3 of concrete?

- A) 54 ft^3
- B) 60 ft^3
- C) 84 ft^3
- D) 108 ft^3

4

If Andy drove 84 miles in 1 hour 45 minutes, how many miles can he drive in 5 hours?

5

A collection of quarters, dimes, and nickels is worth \$5.00. If the ratio of quarters to dimes to nickels is $2 : 4 : 7$, how many quarters are there?

6

If $\frac{5x}{3} = \frac{x+14}{2}$, what is the value of x ?

7

A trail mix contains raisin, peanut, and chocolate. The ratio of raisin to peanut is $2 : 3$ and the ratio of peanut to chocolate is $5 : 8$. What is the ratio of raisin to chocolate?

Exercises - Ratios, Rates, and Proportions Word Problems

1

If 20 machines produce 1,240 printers in a day, how many more machines are needed to produce 1,984 printers in a day?

- A) 12
- B) 20
- C) 24
- D) 32

2

If $\frac{3}{4}$ quart of lemonade concentrate is mixed with $6\frac{2}{3}$ quarts of water to make lemonade for 40 people, how many quarts of lemonade concentrate are needed to make the lemonade for 24 people?

- A) $\frac{3}{10}$
- B) $\frac{7}{20}$
- C) $\frac{2}{5}$
- D) $\frac{9}{20}$

3

A machine produced 735 tapes in $5\frac{1}{4}$ hours. What fraction of the 735 tapes was produced in one hour?

- A) $\frac{1}{7}$
- B) $\frac{4}{21}$
- C) $\frac{5}{21}$
- D) $\frac{2}{7}$

4

A 32-acre field yields 768 bushels of corn each year. How many more acres are needed to yield 960 bushels of corn each year?

- A) 6
- B) 8
- C) 10
- D) 12

5

The length of a rectangle is 8 inches longer than the width. If the ratio of the length to perimeter is 5:16, what is the area of the rectangle?

- A) 160 in^2
- B) 180 in^2
- C) 240 in^2
- D) 280 in^2

6

If 12 grams of coffee costs x dollars and each gram makes y cups of coffee, what is the cost of one cup of coffee in terms of x and y ?

- A) $\frac{12y}{x}$
- B) $\frac{y}{12x}$
- C) $\frac{12x}{y}$
- D) $\frac{x}{12y}$

Answer Key

Section 6-1

1. B 2. D 3. C 4. D 5. 23
6. 5 7. 32.4

Section 6-2

1. D 2. B 3. B 4. 240 5. 8
6. 6 7. $\frac{5}{12}$

Section 6-3

1. A 2. D 3. B 4. B 5. C
6. D

Chapter 6 Practice Test

1. C 2. A 3. C 4. C 5. B
6. A 7. A 8. B 9. B 10. D
11. $\frac{25}{4}$ or 6.25 12. 3

Answers and Explanations**Section 6-1**

1. B

$$\frac{1\frac{3}{4}}{2\frac{1}{2}} = \frac{14}{x}$$

The ratio of $1\frac{3}{4}$ to $2\frac{1}{2}$ is equal to the ratio of 14 to x .

$$1\frac{3}{4} \cdot x = 14 \cdot 2\frac{1}{2}$$

Cross Products

$$\frac{7}{4}x = 14 \cdot \frac{5}{2}$$

Simplify.

$$\frac{7}{4}x = 35$$

Simplify.

$$\frac{4}{7} \cdot \frac{7}{4}x = \frac{4}{7} \cdot 35$$

Multiply each side by $\frac{4}{7}$.

$$x = 20$$

Simplify.

2. D

Let x and y be the two numbers.

$$x + y = 14$$

The sum of two numbers is 14.

$$\frac{x}{y} = -3$$

The ratio of the two numbers is -3 .

$$\frac{x}{y} = -3 \Rightarrow x = -3y$$

$$\begin{aligned} x + y &= 14 && \text{First equation} \\ (-3y) + y &= 14 && \text{Substitute } -3y \text{ for } x. \\ -2y &= 14 && \text{Simplify.} \\ y &= -7 \end{aligned}$$

Substitute $y = -7$ in the first equation.

$$x + (-7) = 14 \Rightarrow x = 21$$

Therefore the product of the two numbers is

$$x \cdot y = 21 \cdot (-7) = -147.$$

3. C

$$\begin{aligned} 2(x - y) &= 3y \\ 2x - 2y &= 3y && \text{Distributive property} \\ 2x &= 5y && \text{Add } 2y \text{ to each side.} \end{aligned}$$

$$\frac{2x}{2} = \frac{5y}{2}$$

Divide each side by 2.

$$x = \frac{5}{2}y$$

Simplify.

$$\frac{x}{y} = \frac{\frac{5}{2}y}{y}$$

Divide each side by y .

$$\frac{x}{y} = \frac{5}{2}$$

Simplify.

4. D

Let $6x$ = the length and $7x$ = the width of the rectangle.

$$P = 2\ell + 2w$$

Perimeter of a rectangle.

$$78 = 2(6x) + 2(7x) \quad P = 78, \ell = 6x, \text{ and } w = 7x$$

$$78 = 26x$$

Simplify.

$$3 = x$$

Divide each side by 26.

Therefore, the length of the rectangle is $6 \cdot 3$ or 18, and the width of the rectangle is $7 \cdot 3$ or 21. The area of the rectangle is $18 \cdot 21$ or 378.

5. 23

$$\text{Gas Mileage} = \frac{\text{Number of Miles Traveled}}{\text{Number of Gallons of Gas Used}}$$

$$= \frac{218.5}{9.5} = 23$$

The car's gas mileage is 23 miles per gallon.

6. 5

$$\text{Unit Price} = \frac{\text{Price of Package}}{\text{Number of Units in the Package}}$$

$$= \frac{0.95}{20} = 0.0475$$

The unit price of the vitamin water to the nearest cent is 5.

7. 32.4

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\frac{2.7 \text{ grams}}{1 \text{ cm}^3} = \frac{x \text{ grams}}{12 \text{ cm}^3}$$

$$x = 2.7 \times 12 = 32.4 \text{ grams}$$

Section 6-2

1. D

Set up a proportion.

$$\frac{1 \text{ inch}}{5 \text{ miles}} = \frac{10 \text{ inches}}{x \text{ miles}} \Rightarrow x = 50 \text{ miles}$$

$$\frac{1 \text{ inch}}{5 \text{ miles}} = \frac{7.2 \text{ inches}}{y \text{ miles}} \Rightarrow y = 7.2 \times 5 = 36 \text{ miles}$$

The area of the state is 50×36 , or $1,800 \text{ mi}^2$.

2. B

Let the number of students = $27x$ and let the number of teachers = $2x$. Then, there will be $27x + 2x$, or $29x$ students and teachers who are in the meeting. Now set up a proportion.

$$\frac{\text{total in the meeting}}{\text{number of teachers}} = \frac{29x}{2x} \Rightarrow$$

$$\frac{754}{\text{number of teachers}} = \frac{29\cancel{x}}{2\cancel{x}} \Rightarrow$$

$$\text{number of teachers} = \frac{754 \times 2}{29} = 52$$

3. B

Let $5x$ = the volume of cement, $9x$ = the volume of sand, and $13x$ = the volume of gravel. Thus the total volume of concrete is $5x + 9x + 13x$, or $27x$, which is equal to 324 ft^3 .

$$27x = 324 \Rightarrow x = 12$$

Therefore, the amount of cement is $5x = 5 \cdot 12 = 60$.

4. 240

$$1 \text{ hour } 45 \text{ minutes} = 1\frac{3}{4} \text{ hours}$$

Set up a proportion.

$$\frac{84 \text{ miles}}{1\frac{3}{4} \text{ hours}} = \frac{x \text{ miles}}{5 \text{ hours}}$$

$$1\frac{3}{4}x = 84 \cdot 5 \quad \text{Cross Products}$$

$$\frac{7}{4}x = 420 \quad \text{Simplify.}$$

$$\frac{4}{7} \cdot \frac{7}{4}x = \frac{4}{7} \cdot 420 \quad \text{Multiply each side by } \frac{4}{7}.$$

$$x = 240$$

He can drive 240 miles in 5 hours.

5. 8

Let $2x$ = the number of quarters, $4x$ = the number of dimes, and $7x$ = the number of nickels.

Then the total amount in terms of x , $2x(0.25) + 4x(0.1) + 7x(0.05)$, is equal to \$5.00.

$$2x(0.25) + 4x(0.1) + 7x(0.05) = 5.00$$

$$0.5x + 0.4x + 0.35x = 5$$

$$1.25x = 5$$

$$x = 4$$

There are $2x = 2 \cdot 4$, or 8 quarters.

6. 6

$$\frac{5x}{3} = \frac{x+14}{2}$$

$$2(5x) = 3(x+14) \quad \text{Cross Products}$$

$$10x = 3x + 42$$

$$7x = 42$$

$$x = 6$$

7. $\frac{5}{12}$

Let r = the amount of raisin, p = the amount of peanut, and c = the amount of chocolate. Then

$$\frac{r}{p} = \frac{2}{3} \quad \text{The ratio of raisin to peanut is } 2:3.$$

$$3r = 2p \quad \text{Cross Products}$$

$$p = \frac{3}{2}r \quad \text{Solve for } p.$$

$$\frac{p}{c} = \frac{5}{8} \quad \text{The ratio of peanut to chocolate is } 5:8.$$

$$8p = 5c \quad \text{Cross Products}$$

$$p = \frac{5}{8}c \quad \text{Solve for } p.$$

Equate the two equations solved for p .

$$\frac{3}{2}r = \frac{5}{8}c \Rightarrow \frac{2}{3} \cdot \frac{3}{2}r = \frac{2}{3} \cdot \frac{5}{8}c$$

$$\Rightarrow r = \frac{5}{12}c \Rightarrow \frac{r}{c} = \frac{5}{12}$$

Section 6-3

1. A

Set up a proportion.

$$\frac{20}{1240} = \frac{x}{1984} \quad \begin{array}{l} \leftarrow \text{number of machines} \\ \leftarrow \text{number of printers} \end{array}$$

$$1240x = 20 \cdot 1984 \quad \text{Cross Products}$$

$$x = \frac{20 \cdot 1984}{1240} = 32$$

Altogether we need 32 machines, therefore we need $32 - 20$, or 12, more machines.

2. D

Let x = the number of quarts of lemonade concentrate needed for 24 people.

In this question “ $6\frac{2}{3}$ quarts of water” was unnecessary information.

$$\frac{\frac{3}{4}}{40} = \frac{x}{24} \quad \begin{array}{l} \leftarrow \text{quarts} \\ \leftarrow \text{people} \end{array}$$

$$40x = 24 \cdot \frac{3}{4} \quad \text{Cross products}$$

$$x = 24 \cdot \frac{3}{4} \cdot \frac{1}{40} = \frac{9}{20}$$

3. B

The number of tapes produced in one hour is equal to $735 \div 5\frac{1}{4}$, or 140.

The fraction of 735 tapes produced in one hour is $\frac{140}{735}$, or $\frac{4}{21}$.

4. B

Set up a proportion.

$$\frac{32}{768} = \frac{x}{960} \quad \begin{array}{l} \leftarrow \text{number of acres} \\ \leftarrow \text{number of bushels} \end{array}$$

$$768x = 32 \cdot 960 \quad \text{Cross products}$$

$$x = \frac{32 \cdot 960}{768} = 40$$

Altogether we need 40 acres, therefore we need $40 - 32$, or 8, more acres.

5. C

Let x = the width of the rectangle, then $x + 8$ = the length of the rectangle.

$$P = 2\ell + 2w$$

Perimeter of a rectangle.

$$P = 2(x + 8) + 2(x)$$

 $\ell = x + 8$, and $w = x$

$$P = 4x + 16$$

Simplify.

$$\frac{\text{length}}{\text{perimeter}} = \frac{x + 8}{4x + 16} = \frac{5}{16}$$

$$16(x + 8) = 5(4x + 16) \quad \text{Cross Products}$$

$$16x + 128 = 20x + 80$$

$$48 = 4x$$

$$12 = x$$

The length of the rectangle is $12 + 8$, or 20 and the width of the rectangle is 12.

The area of the rectangle is $20 \cdot 12$, or 240.

6. D

If 12 grams of coffee cost x dollars, the cost

of each gram of coffee is $\frac{x}{12}$ dollars. Let one

cup of coffee cost d dollars, and set up a proportion to find the cost of one cup of coffee.

$$\frac{\frac{x}{12}}{y} = \frac{d}{1} \quad \begin{array}{l} \leftarrow \text{cost in dollars} \\ \leftarrow \text{number of cups} \end{array}$$

$$y \cdot d = \frac{x}{12} \quad \text{Cross Products}$$

$$d = \frac{x}{12y}$$

Chapter 6 Practice Test

1. C

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ m}^3 = (100 \text{ cm})^3 = 1,000,000 \text{ cm}^3$$

$$0.01 \text{ m}^3 = 0.01 \times 1,000,000 \text{ cm}^3 = 10,000 \text{ cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$4.54 \text{ grams/cm}^3 = \frac{\text{Mass}}{0.01 \text{ m}^3} = \frac{\text{Mass}}{10,000 \text{ cm}^3}$$

$$\begin{aligned} \text{Mass} &= 4.54 \frac{\text{grams}}{\text{cm}^3} \cdot 10,000 \text{ cm}^3 \\ &= 45,400 \text{ grams} \end{aligned}$$

2. A

$$\text{Total time} = \text{Jason's time} + \text{Donny's time}$$

$$= 4 \text{ hour } 25 \text{ min} + 2 \text{ hour } 15 \text{ min}$$

$$= 4\frac{5}{12} \text{ hour} + 2\frac{1}{4} \text{ hour} = 6\frac{2}{3} \text{ hour}$$

The amount Donny received

$$= 1,200 \times \frac{2\frac{1}{2} \text{ hour}}{6\frac{2}{3} \text{ hour}} = 1,200 \cdot \frac{\frac{9}{2}}{\frac{20}{3}} = 1,200 \cdot \frac{9}{4} \cdot \frac{3}{20}$$

$$= 405$$

3. C

If the ratio of white balls to yellow balls is $\frac{3}{10}$,

$3n$ represents the number of white balls and $10n$ represents the number of yellow balls (n is a positive integer).

Since the total number of balls in the bag is $3n + 10n$, or $13n$, and n is a positive integer, the number of balls will be a multiple of 13.

Choice C is correct, because 42 is not a multiple of 13.

4. C

Let m = the number of miles traveled in y minutes. Substitute 60 minutes for 1 hour and set up a proportion.

$$\frac{x}{60} = \frac{m}{y} \quad \leftarrow \begin{array}{l} \text{number of miles} \\ \text{number of minutes} \end{array}$$

$$60m = xy \quad \text{Cross Products}$$

$$m = \frac{xy}{60}$$

5. B

Let x = the number of years it will take the tree to reach a height of 30 feet.

Also, 8 inches = $\frac{8}{12}$ feet.

$$8 + \frac{8}{12}x = 30 \quad \begin{array}{l} \text{The tree is 8 feet tall and will} \\ \text{grow } \frac{8}{12}x \text{ feet in } x \text{ years.} \end{array}$$

$$\frac{8}{12}x = 22$$

$$x = 22 \cdot \frac{12}{8} = 33$$

6. A

m minutes = $60m$ seconds

Let p = the number of pages he reads in $20m$ seconds.

Set up a proportion.

$$\frac{x}{60m} = \frac{p}{20m} \quad \leftarrow \begin{array}{l} \text{number of pages} \\ \text{number of seconds} \end{array}$$

$$60m \cdot p = 20m \cdot x$$

Cross Products

$$p = \frac{20m \cdot x}{60m} = \frac{1}{3}x$$

7. A

$$\frac{x}{y} = 1$$

$$y \cdot \frac{x}{y} = y \cdot 1$$

Multiply each side by y .

$$x = y$$

Simplify.

$$x - y = y - y$$

Subtract y from each side.

$$x - y = 0$$

Simplify.

$$x - y - 1 = 0 - 1$$

Subtract 1 from each side.

$$x - y - 1 = -1$$

Simplify.

8. B

Let m = the number of males in the room and let f = the number of females in the room.

$$\frac{m}{f} = \frac{4}{5}$$

The ratio of males to females is 4 to 5.

$$5m = 4f$$

Cross Products

$$\frac{m+8}{f} = \frac{6}{5}$$

After 8 males enter the room, the ratio of males to females is 6 to 5.

$$5(m+8) = 6f$$

Cross Products

$$5m + 40 = 6f$$

Simplify.

$$4f + 40 = 6f$$

Substitute $4f$ for $5m$.

$$40 = 2f$$

Subtract $2f$ from each side.

$$20 = f$$

Divide each side by 2.

Substituting 20 for f in the equation $5m = 4f$ gives $5m = 4 \cdot 20$. Solving for m yields $m = 16$.

The total number of people in the room before the additional males enter the room is

$$m + f = 16 + 20 = 36.$$

9. B

If a person is born every 5 seconds, 12 people are born per minute. If a person dies every 12 seconds, 5 people die per minute. Every minute the population grows by $12 - 5$, or 7, people.

Therefore, it takes $\frac{60}{7}$ seconds, or $8\frac{4}{7}$ seconds,

for the population to grow by one person.

10. D

Total area of the wall = $9 \times 12 = 108 \text{ ft}^2$.Let it take p gallons of paint to paint 108 ft^2 .

Set up a proportion.

$$\frac{1}{s} = \frac{p}{108} \quad \begin{array}{l} \leftarrow \text{number of gallons} \\ \leftarrow \text{number of square feet} \end{array}$$

$$sp = 108 \quad \text{Cross Products}$$

$$p = \frac{108}{s}$$

It takes $\frac{108}{s}$ gallons of paint to paint 108 ft^2 .Since each gallon costs g dollars, the total costwill be $\frac{108}{s} \cdot g$ dollars.11. $\frac{25}{4}$ or 6.25

$$2 \text{ in} = 5 \text{ cm}$$

$$1 \text{ in} = \frac{5}{2} \text{ cm} \quad \text{Divide each side by 2.}$$

$$(1 \text{ in})^2 = \left(\frac{5}{2} \text{ cm}\right)^2 \quad \text{Square both sides.}$$

$$1 \text{ in}^2 = \frac{25}{4} \text{ cm}^2 \quad \text{Simplify.}$$

There are $\frac{25}{4}$ square centimeters in 1 square inch.

12. 3

The reduced length of the painting is $18 - x$ and the reduced width of the painting is $12 - x$.

$$\frac{18 - x}{12 - x} = \frac{5}{3} \quad \text{The new ratio is 5 to 3.}$$

$$3(18 - x) = 5(12 - x) \quad \text{Cross Products}$$

$$54 - 3x = 60 - 5x \quad \text{Distributive Property}$$

$$54 + 2x = 60 \quad \text{Add } 5x \text{ to each side.}$$

$$2x = 6 \quad \text{Subtract 54 from each side.}$$

$$x = 3 \quad \text{Divide each side by 2.}$$

