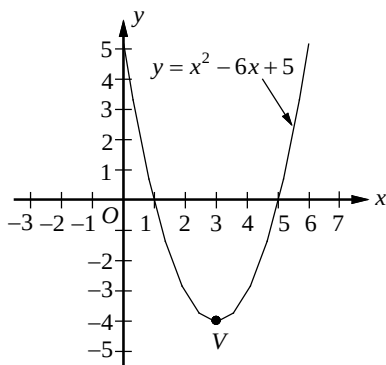


Exercises - Graphs of Quadratic Equations

Questions 1 and 2 refer to the following information.



The graph of quadratic function $y = x^2 - 6x + 5$ is shown above.

1

Which of the following is an equivalent form of the equation of the graph shown above, from which the coordinates of vertex V can be identified as constants in the equation?

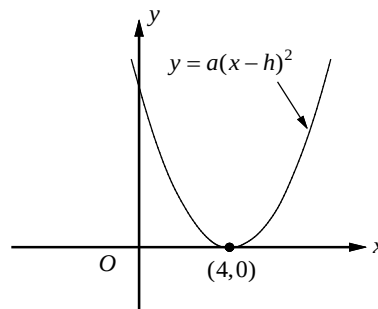
- A) $y = (x - 1)(x - 5)$
- B) $y = (x + 1)(x + 5)$
- C) $y = x(x - 6) + 5$
- D) $y = (x - 3)^2 - 4$

2

Which of the following is an equivalent form of the equation of the graph shown above, that displays the x -intercepts of the parabola as constants?

- A) $y = (x - 1)(x - 5)$
- B) $y = (x + 1)(x + 5)$
- C) $y = x(x - 6) + 5$
- D) $y = (x - 3)^2 - 4$

3



In the xy -plane above, the parabola $y = a(x - h)^2$ has one x -intercept at $(4, 0)$. If the y -intercept of the parabola is 9, what is the value of a ?

4

In the xy -plane, if the parabola with equation $y = a(x + 2)^2 - 15$ passes through $(1, 3)$, what is the value of a ?

5

The graph of the equation $y = a(x - 1)(x + 5)$ is a parabola with vertex (h, k) . If the minimum value of y is -12 , what is the value of a ?

Exercises – Factoring Trinomials

1

$$x^2 - 2x - 24$$

Which of the following is equivalent to the expression above?

- A) $(x+3)(x-8)$
- B) $(x-3)(x+8)$
- C) $(x-6)(x+4)$
- D) $(x+6)(x-4)$

2

$$x^2 - 17x + 72$$

Which of the following is equivalent to the expression above?

- A) $(x+8)(x-9)$
- B) $(x-8)(x-9)$
- C) $(x-12)(x-6)$
- D) $(x-12)(x+6)$

3

$$-x^2 + 5x + 84$$

Which of the following is equivalent to the expression above?

- A) $(12-x)(x+7)$
- B) $(12+x)(x-7)$
- C) $(21+x)(x-4)$
- D) $(21-x)(x+4)$

4

$$3x^2 + 7x - 6$$

Which of the following is equivalent to the expression above?

- A) $(3x-2)(x+3)$
- B) $(3x+2)(x-3)$
- C) $(3x-1)(x+6)$
- D) $(3x+1)(x-6)$

5

$$2x^2 + x - 15$$

Which of the following is equivalent to the expression above?

- A) $(2x+3)(x-5)$
- B) $(2x-3)(x+5)$
- C) $(2x-5)(x+3)$
- D) $(2x+5)(x-3)$

6

$$-6x^2 + x + 2$$

Which of the following is equivalent to the expression above?

- A) $-(6x+1)(x-2)$
- B) $-(6x-1)(x+2)$
- C) $-(3x+2)(2x-1)$
- D) $-(3x-2)(2x+1)$

Exercises – Factoring Differences of Squares and Perfect Square Trinomials

1

$$3x^2 - 48$$

Which of the following is equivalent to the expression above?

- A) $3(x-4)(x+4)$
- B) $3(x-4)^2$
- C) $(3x-4)(x+4)$
- D) $(3x+4)(x-4)$

2

$$x - 6\sqrt{x} - 16$$

Which of the following is equivalent to the expression above?

- A) $(\sqrt{x} - 4)^2$
- B) $(\sqrt{x} - 4)(\sqrt{x} + 4)$
- C) $(\sqrt{x} + 8)(\sqrt{x} - 2)$
- D) $(\sqrt{x} - 8)(\sqrt{x} + 2)$

3

If $x^2 + y^2 = 10$ and $xy = -3$, what is the value of $(x - y)^2$?

- A) 12
- B) 16
- C) 20
- D) 25

4

If $x + y = 10$ and $x - y = 4$, what is the value of $x^2 - y^2$?

- A) 20
- B) 24
- C) 36
- D) 40

5

$$6x^2 + 7x - 24 = 0$$

If r and s are two solutions of the equation above and $r > s$, which of the following is the value of $r - s$?

- A) $\frac{7}{6}$
- B) $\frac{16}{3}$
- C) $\frac{25}{6}$
- D) $\frac{20}{3}$

6

$$x^2 - 3x = 28$$

If r and s are two solutions of the equation above, which of the following is the value of $r + s$?

- A) -3
- B) 3
- C) 6
- D) 9

Exercises - Solving Quadratic Equations by Completing the Square

1

If $x^2 - 10x = 75$ and $x < 0$, what is the value of $x + 5$?

- A) -15
- B) -10
- C) -5
- D) 0

2

If $x^2 - kx = 20$ and $x - \frac{k}{2} = 6$, which of the following is a possible value of x ?

- A) 2
- B) 4
- C) 6
- D) 8

3

$$x^2 - \frac{k}{3}x = 5$$

Which of the following is an equivalent form of the equation shown above, from which the equation could be solved by completing the square?

- A) $x^2 - \frac{k}{3}x + \frac{k}{6} = \frac{k}{6} + 5$
- B) $x^2 - \frac{k}{3}x + \frac{k^2}{9} = \frac{k^2}{9} + 5$
- C) $x^2 - \frac{k}{3}x + \frac{k^2}{36} = \frac{k^2}{36} + 5$
- D) $x^2 - \frac{k}{3}x + \frac{k^2}{6} = \frac{k^2}{6} + 5$

4

$$x^2 - rx = \frac{k^2}{4}$$

In the quadratic equation above, k and r are constants. What are the solutions for x ?

- A) $x = \frac{r}{4} \pm \frac{\sqrt{k^2 + 2r^2}}{4}$
- B) $x = \frac{r}{2} \pm \frac{\sqrt{k^2 + 8r^2}}{4}$
- C) $x = \frac{r}{4} \pm \frac{\sqrt{k^2 + r^2}}{2}$
- D) $x = \frac{r}{2} \pm \frac{\sqrt{k^2 + r^2}}{2}$

5

If $(x-7)(x-s) = x^2 - rx + 14$ for all values of x , what is the value of $r + s$?

6

If $x^2 - \frac{3}{2}x + c = (x-k)^2$, what is the value of c ?

Exercises - Quadratic Formula and the Discriminant

1

$$(p-1)x^2 - 2x - (p+1) = 0$$

In the quadratic equation above, p is a constant.
What are the solutions for x ?

- A) $\frac{1+\sqrt{2-p^2}}{p-1}$ and $\frac{1-\sqrt{2-p^2}}{p-1}$
- B) $\frac{1+2p}{p-1}$ and -1
- C) $\frac{p+1}{p-1}$ and -1
- D) $\frac{p+1}{p-1}$ and $\frac{2p+1}{p-1}$

2

What is the sum of all values of x that satisfy $3x^2 + 12x - 29 = 0$?

- A) -4
- B) -2
- C) 2
- D) 4

3

If the quadratic equation $kx^2 + 6x + 4 = 0$ has exactly one solution, what is the value of k ?

- A) $\frac{3}{2}$
- B) $\frac{5}{2}$
- C) $\frac{7}{4}$
- D) $\frac{9}{4}$

4

$$\begin{cases} y = bx - 3 \\ y = ax^2 - 7x \end{cases}$$

In the system of equations above, a and b are constants. For which of the following values of a and b does the system of equations have exactly two real solutions?

- A) $a = 3, b = -2$
- B) $a = 5, b = 0$
- C) $a = 7, b = 2$
- D) $a = 9, b = 4$

5

What are the solutions to $x^2 + 4 = -6x$?

- A) $-3 \pm \sqrt{13}$
- B) $-3 \pm \sqrt{5}$
- C) $-6 \pm \sqrt{5}$
- D) $-6 \pm \sqrt{13}$

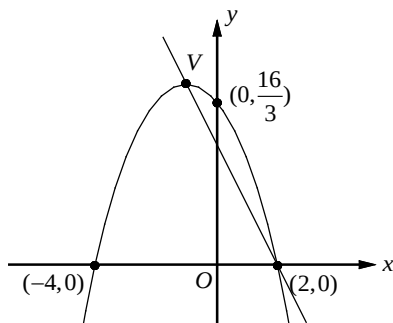
6

Which of the following equations has no real solution?

- A) $5x^2 - 10x = 6$
- B) $4x^2 + 8x + 4 = 0$
- C) $3x^2 - 5x = -3$
- D) $-\frac{1}{3}x^2 + 2x - 2 = 0$

Exercises - Solving Systems Consisting Linear and Quadratic Equations

1



The xy -plane above shows two x -intercepts, a y -intercept and vertex V of a parabola. If the line passes through the points $(2, 0)$ and V , which of the following must be the y -intercept of the line?

- A) 3
- B) $\frac{7}{2}$
- C) 4
- D) $\frac{9}{2}$

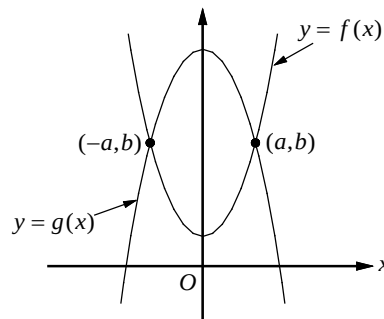
2

$$\begin{cases} y = x^2 + x \\ y = ax - 1 \end{cases}$$

In the system of equations above, $a > 0$. If the system of equations has exactly one real solution, what is the value of a ?

- A) $\frac{5}{2}$
- B) 3
- C) $\frac{7}{2}$
- D) 4

3



The function f and g , defined by $f(x) = 2x^2 + 2$ and $g(x) = -2x^2 + 18$, are graphed in the xy -plane above. The two graphs intersect at the points (a, b) and $(-a, b)$. What is the value of b ?

- A) 6
- B) 8
- C) 10
- D) 12

4

$$\begin{cases} x^2 + y^2 = 14 \\ x^2 - y = 2 \end{cases}$$

If (x, y) is a solution to the system of equations above, what is the value of x^2 ?

- A) 2
- B) 3
- C) 4
- D) 5

Answer Key

Section 11-1

1. D 2. A 3. $\frac{9}{16}$ 4. 2 5. $\frac{4}{3}$

Section 11-2

1. C 2. B 3. A 4. A 5. C
6. D

Section 11-3

1. A 2. D 3. B 4. D 5. C
6. B

Section 11-4

1. D 2. A 3. C 4. D 5. 11
6. $\frac{9}{16}$

Section 11-5

1. C 2. A 3. D 4. D 5. B
6. C

Section 11-6

1. C 2. B 3. C 4. D

Chapter 11 Practice Test

1. B 2. C 3. A 4. A 5. D
6. C 7. A 8. B 9. B

Answers and Explanations**Section 11-1**

1. D

Change the given equation into the vertex form $y = a(x-h)^2 + k$, in which (h, k) is the vertex of the parabola, by completing the square.

$$\begin{aligned} y &= x^2 - 6x + 5 \\ &= x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 + 5 \\ &= (x^2 - 6x + 9) - 9 + 5 \\ &= (x-3)^2 - 4 \end{aligned}$$

The coordinate of the vertex can be read as $(3, -4)$.

2. A

Change the given equation into the factored form $y = (x-a)(x-b)$, in which $x = a$ and $x = b$ are the x -intercepts of the parabola. Find two numbers with a sum of -6 and a product of 5 . The two numbers are -1 and -5 .

$y = x^2 - 6x + 5$ can be written in the factored form $y = (x-1)(x-5)$. The x -intercepts are 1 and 5 .

3. $\frac{9}{16}$

$$y = a(x-h)^2$$

$$0 = a(4-h)^2 \quad x\text{-intercept at } (4, 0)$$

Since $a \neq 0$, $4-h = 0$, or $h = 4$.

The graph of the parabola passes through $(0, 9)$, since the y -intercept of the parabola is 9 .

$$9 = a(0-h)^2 \quad y\text{-intercept at } (0, 9)$$

$$9 = ah^2 \quad \text{Simplify.}$$

$$9 = a(4)^2 \quad \text{Substitute 4 for } h.$$

$$\frac{9}{16} = a$$

4. 2

$$y = a(x+2)^2 - 15$$

$$3 = a(1+2)^2 - 15 \quad x=1 \text{ and } y=3$$

$$3 = 9a - 15$$

$$18 = 9a$$

$$2 = a$$

5. $\frac{4}{3}$

The x -intercepts of the graph of the equation $y = a(x-1)(x+5)$ are -5 and 1 . The x -coordinate of the vertex is the average of the two x -intercepts.

Therefore, $h = \frac{-5+1}{2} = -2$. The value of k is -12

because the minimum value of y is -12 . So the coordinate of the vertex is $(-2, -12)$. Substitute $x = -2$ and $y = -12$ in the given equation.

$$-12 = a(-2-1)(-2+5)$$

$$-12 = -9a$$

$$\frac{12}{9} = a \text{ or } a = \frac{4}{3}$$

Section 11-2

1. C

$$x^2 - 2x - 24$$

Find two numbers with a sum of -2 and a product of -24 . The two numbers are -6 and 4 .

$$\text{Therefore, } x^2 - 2x - 24 = (x - 6)(x + 4).$$

2. B

$$x^2 - 17x + 72$$

Find two numbers with a sum of -17 and a product of 72 . The two numbers are -8 and -9 .

$$\text{Therefore, } x^2 - 17x + 72 = (x - 8)(x - 9).$$

3. A

$$-x^2 + 5x + 84 = -(x^2 - 5x - 84)$$

Find two numbers with a sum of -5 and a product of -84 . The two numbers are -12 and 7 .

$$\begin{aligned} -x^2 + 5x + 84 &= -(x^2 - 5x - 84) \\ &= -(x - 12)(x + 7) = (12 - x)(x + 7) \end{aligned}$$

4. A

$$3x^2 + 7x - 6$$

Find two numbers with a sum of 7 and a product of $3 \cdot -6$ or -18 . The two numbers are -2 and 9 .

$$\begin{aligned} 3x^2 + 7x - 6 &= 3x^2 - 2x + 9x - 6 && \text{Write } 7x \text{ as } -2x + 9x. \\ &= (3x^2 - 2x) + (9x - 6) && \text{Group terms.} \\ &= x(3x - 2) + 3(3x - 2) && \text{Factor out the GCF.} \\ &= (3x - 2)(x + 3) && \text{Distributive Property} \end{aligned}$$

5. C

$$2x^2 + x - 15$$

Find two numbers with a sum of 1 and a product of $2 \cdot -15$ or -30 . The two numbers are -5 and 6 .

$$\begin{aligned} 2x^2 + x - 15 &= 2x^2 - 5x + 6x - 15 && \text{Write } x \text{ as } -5x + 6x. \\ &= (2x^2 - 5x) + (6x - 15) && \text{Group terms.} \\ &= x(2x - 5) + 3(2x - 5) && \text{Factor out the GCF.} \\ &= (2x - 5)(x + 3) && \text{Distributive Property} \end{aligned}$$

6. D

$$-6x^2 + x + 2 = -(6x^2 - x - 2)$$

Find two numbers with a sum of -1 and a product of $6 \cdot -2$ or -12 . The two numbers are -4 and 3 .

$$\begin{aligned} -6x^2 + x + 2 &= -(6x^2 - x - 2) \\ &= -(6x^2 - 4x + 3x - 2) && \text{Write } -x \text{ as } -4x + 3x. \\ &= -[(6x^2 - 4x) + (3x - 2)] && \text{Group terms.} \\ &= -[2x(3x - 2) + (3x - 2)] && \text{Factor out the GCF.} \\ &= -(3x - 2)(2x + 1) && \text{Distributive Property} \end{aligned}$$

Section 11-3

1. A

$$\begin{aligned} 3x^2 - 48 &= 3(x^2 - 16) && \text{Factor out the GCF.} \\ &= 3((x)^2 - (4)^2) && \text{Write in the form } a^2 - b^2. \\ &= 3(x - 4)(x + 4) && \text{Difference of Squares} \end{aligned}$$

2. D

$$\begin{aligned} x - 6\sqrt{x} - 16 & \text{Let } y = \sqrt{x}, \text{ then } y^2 = x. \\ x - 6\sqrt{x} - 16 &= y^2 - 6y - 16 && y = \sqrt{x} \text{ and } y^2 = x \\ &= (y - 8)(y + 2) \\ &= (\sqrt{x} - 8)(\sqrt{x} + 2) && y = \sqrt{x} \text{ and } y^2 = x \end{aligned}$$

3. B

$$\begin{aligned} (x - y)^2 &= (x - y)(x - y) \\ &= x^2 - 2xy + y^2 \\ &= (x^2 + y^2) - 2xy \\ &= 10 - 2(-3) = 16 && x^2 + y^2 = 10 \text{ and } xy = -3 \end{aligned}$$

4. D

$$\begin{aligned} x^2 - y^2 &= (x + y)(x - y) \\ &= (10)(4) \\ &= 40 && x + y = 10 \text{ and } x - y = 4 \end{aligned}$$

5. C

$$6x^2 + 7x - 24 = 0$$

$$(3x+8)(2x-3) = 0 \quad \text{Factor.}$$

$$3x+8=0 \text{ or } 2x-3=0 \quad \text{Zero Product Property}$$

$$x = -\frac{8}{3} \text{ or } x = \frac{3}{2} \quad \text{Solve each equation.}$$

$$\text{Since } \frac{3}{2} > -\frac{8}{3}, r = \frac{3}{2} \text{ and } s = -\frac{8}{3}.$$

$$r-s = \frac{3}{2} - \left(-\frac{8}{3}\right) = \frac{9}{6} + \frac{16}{6} = \frac{25}{6}$$

6. B

$$x^2 - 3x = 28$$

$$x^2 - 3x - 28 = 0 \quad \text{Make one side 0.}$$

$$(x-7)(x+4) = 0 \quad \text{Factor.}$$

$$x-7=0 \text{ or } x+4=0 \quad \text{Zero Product Property}$$

$$x=7 \text{ or } x=-4 \quad \text{Solve each equation.}$$

$$\text{Therefore, } r+s = 7+(-4) = 3.$$

Section 11-4

1. D

$$x^2 - 10x = 75$$

$$\text{Add } \left(\frac{-10}{2}\right)^2 \text{ to each side.}$$

$$x^2 - 10x + \left(-\frac{10}{2}\right)^2 = 75 + \left(-\frac{10}{2}\right)^2$$

$$x^2 - 10x + 25 = 75 + 25 \quad \text{Simplify.}$$

$$(x-5)^2 = 100 \quad \text{Factor } x^2 - 10x + 25.$$

$$x-5 = \pm 10 \quad \text{Take the square root.}$$

$$x = 5 \pm 10 \quad \text{Add 5 to each side.}$$

$$x = 5+10 \text{ or } x = 5-10 \quad \text{Separate the solutions.}$$

$$x = 15 \text{ or } x = -5 \quad \text{Simplify.}$$

$$\text{If } x < 0, x = -5. \text{ Therefore, } x+5 = -5+5 = 0.$$

2. A

$$x^2 - kx = 20$$

$$\text{Add } \left(\frac{-k}{2}\right)^2 \text{ to each side.}$$

$$x^2 - kx + \left(\frac{-k}{2}\right)^2 = 20 + \left(\frac{-k}{2}\right)^2$$

$$x^2 - kx + \frac{k^2}{4} = 20 + \frac{k^2}{4} \quad \text{Simplify.}$$

$$\left(x - \frac{k}{2}\right)^2 = 20 + \frac{k^2}{4} \quad \text{Factor } x^2 - kx + \frac{k^2}{4}.$$

$$(6)^2 = 20 + \frac{k^2}{4} \quad \text{Substitute 6 for } x - \frac{k}{2}.$$

$$16 = \frac{k^2}{4}$$

$$\text{Solving for } k \text{ gives } k = \pm 8.$$

$$\text{Solving the given equation } x - \frac{k}{2} = 6 \text{ for } x$$

$$\text{gives } x = 6 + \frac{k}{2}.$$

$$\text{If } k = 8, x = 6 + \frac{k}{2} = 6 + \frac{8}{2} = 10.$$

$$\text{If } k = -8, x = 6 + \frac{k}{2} = 6 + \frac{-8}{2} = 2.$$

Of the answer choices, 2 is a possible value of x . Therefore, Choice A is correct.

3. C

$$x^2 - \frac{k}{3}x = 5$$

The equation could be solved by completing the square by adding $\left(\frac{1}{2} \cdot \frac{k}{3}\right)^2$, or $\frac{k^2}{36}$, to each side.

Choice C is correct.

4. D

$$x^2 - rx = \frac{k^2}{4}$$

$$\text{Add } \left(\frac{-r}{2}\right)^2, \text{ or } \frac{r^2}{4}, \text{ to each side.}$$

$$x^2 - rx + \frac{r^2}{4} = \frac{k^2}{4} + \frac{r^2}{4}$$

$$\left(x - \frac{r}{2}\right)^2 = \frac{k^2 + r^2}{4} \quad \text{Factor } x^2 - rx + \frac{r^2}{4}.$$

$$x - \frac{r}{2} = \pm \sqrt{\frac{k^2 + r^2}{4}} \quad \text{Take the square root.}$$

$$x - \frac{r}{2} = \pm \frac{\sqrt{k^2 + r^2}}{2} \quad \text{Simplify.}$$

$$x = \frac{r}{2} \pm \frac{\sqrt{k^2 + r^2}}{2} \quad \text{Add } \frac{r}{2} \text{ to each side.}$$

Choice D is correct.

5. 11

$$(x-7)(x-s) = x^2 - rx + 14$$

$$x^2 - (s+7)x + 7s = x^2 - rx + 14$$

Since the x -terms and constant terms have to be equal on both sides of the equation,

$r = s + 7$ and $7s = 14$.
 Solving for s gives $s = 2$.
 $r = s + 7 = 2 + 7 = 9$
 Therefore, $r + s = 9 + 2 = 11$.

6. $\frac{9}{16}$

$$x^2 - \frac{3}{2}x + c = (x - k)^2 \Rightarrow$$

$$x^2 - \frac{3}{2}x + c = x^2 - 2kx + k^2$$

Since the x -terms and constant terms have to be equal on both sides of the equation,

$$2k = \frac{3}{2} \text{ and } c = k^2.$$

$$\text{Solving for } k \text{ gives } k = \frac{3}{4}.$$

$$\text{Therefore, } c = k^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}.$$

Section 11-5

1. C

$$(p-1)x^2 - 2x - (p+1) = 0$$

Use the quadratic formula to find the solutions for x .

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(p-1)(-(p+1))}}{2(p-1)} \\ &= \frac{2 \pm \sqrt{4 + 4(p-1)(p+1)}}{2(p-1)} \\ &= \frac{2 \pm \sqrt{4 + 4p^2 - 4}}{2(p-1)} \\ &= \frac{2 \pm \sqrt{4p^2}}{2(p-1)} = \frac{2 \pm 2p}{2(p-1)} \\ &= \frac{2(1 \pm p)}{2(p-1)} = \frac{1 \pm p}{p-1} \end{aligned}$$

The solutions are $\frac{1+p}{p-1}$ and $\frac{1-p}{p-1}$, or -1 .

Choice C is correct.

2. A

Let r_1 and r_2 be the solutions of the quadratic

$$\text{equation } 3x^2 + 12x - 29 = 0.$$

Use the sum of roots formula.

$$r_1 + r_2 = -\frac{b}{a} = -\frac{12}{3} = -4.$$

3. D

$$kx^2 + 6x + 4 = 0$$

If the quadratic equation has exactly one solution, then $b^2 - 4ac = 0$.

$$b^2 - 4ac = 6^2 - 4(k)(4) = 0 \Rightarrow 36 - 16k = 0$$

$$\Rightarrow k = \frac{36}{16} = \frac{9}{4}$$

4. D

$$y = bx - 3 \text{ and } y = ax^2 - 7x$$

Substitute $bx - 3$ for y in the quadratic equation.

$$bx - 3 = ax^2 - 7x$$

$$ax^2 + (-7 - b)x + 3 = 0 \quad \text{Make one side 0.}$$

The system of equations will have exactly two real solutions if the discriminant of the quadratic equation is positive.

$$(-7 - b)^2 - 4a(3) > 0, \text{ or } (7 + b)^2 - 12a > 0.$$

We need to check each answer choice to find out for which values of a and b the system of equations has exactly two real solutions.

A) If $a = 3$ and $b = -2$, $(7 - 2)^2 - 12(3) < 0$.

B) If $a = 5$ and $b = 0$, $(7 + 0)^2 - 12(5) < 0$.

C) If $a = 7$ and $b = 2$, $(7 + 2)^2 - 12(7) < 0$.

D) If $a = 9$ and $b = 4$, $(7 + 4)^2 - 12(9) > 0$.

Choice D is correct.

5. B

$$x^2 + 4 = -6x$$

$$x^2 + 6x + 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{20}}{2} = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$= -3 \pm \sqrt{5}$$

6. C

If the quadratic equation has no real solution, the discriminant, $b^2 - 4ac$, must be negative. Check each answer choice.

$$\text{A) } 5x^2 - 10x = 6 \Rightarrow 5x^2 - 10x - 6 = 0$$

$$b^2 - 4ac = (-10)^2 - 4(5)(-6) > 0$$

$$\text{B) } 4x^2 + 8x + 4 = 0$$

$$b^2 - 4ac = (8)^2 - 4(4)(4) = 0$$

$$\text{C) } 3x^2 - 5x = -3 \Rightarrow 3x^2 - 5x + 3 = 0$$

$$b^2 - 4ac = (-5)^2 - 4(3)(3) < 0$$

Choice C is correct.

Section 11-6

1. C

Since the two x -intercepts are -4 and 2 , the equation of the parabola can be written as $y = a(x+4)(x-2)$. Substitute $x = 0$ and

$y = \frac{16}{3}$ in the equation, since the graph of the parabola passes through $(0, \frac{16}{3})$.

$$\frac{16}{3} = a(0+4)(0-2)$$

Solving the equation for a gives $a = -\frac{2}{3}$.

Thus the equation of the parabola is

$$y = -\frac{2}{3}(x+4)(x-2).$$

The x -coordinate of the vertex is the average of the two x -intercepts: $\frac{-4+2}{2}$, or -1 .

The y -coordinate of the vertex can be found by substituting -1 for x in the equation of the parabola: $y = -\frac{2}{3}(-1+4)(-1-2) = 6$.

The line passes through $(2, 0)$ and $(-1, 6)$.

The slope of the line is $\frac{6-0}{-1-2} = -2$. The equation of the line in point-slope form is $y - 0 = -2(x - 2)$. To find the y -intercept of the line, substitute 0 for x . $y = -2(0 - 2) = 4$

Choice C is correct.

2. B

$$y = x^2 + x \text{ and } y = ax - 1$$

Substitute $ax - 1$ for y in the quadratic equation.

$$ax - 1 = x^2 + x$$

$$x^2 + (-a + 1)x + 1 = 0 \quad \text{Make one side 0.}$$

If the system of equations has exactly one real solution, the discriminant $b^2 - 4ac$ must be equal to 0 .

$$(-a + 1)^2 - 4(1)(1) = 0 \quad b^2 - 4ac = 0$$

$$a^2 - 2a + 1 - 4 = 0 \quad \text{Simplify.}$$

$$a^2 - 2a - 3 = 0 \quad \text{Simplify.}$$

$$(a - 3)(a + 1) = 0 \quad \text{Factor.}$$

$$a = 3 \text{ or } a = -1 \quad \text{Solutions}$$

Since $a > 0$, $a = 3$.

3. C

One can find the intersection points of the two graphs by setting the two functions $f(x)$ and $g(x)$ equal to one another and then solving for x .

This yields $2x^2 + 2 = -2x^2 + 18$. Adding $2x^2 - 2$ to each side of the equation gives $4x^2 = 16$. Solving for x gives $x = \pm 2$.

$$f(2) = 2(2)^2 + 2 = 10 \text{ and also } f(-2) = 10.$$

The two points of intersections are $(2, 10)$ and $(-2, 10)$. Therefore, the value of b is 10 .

4. D

$$x^2 + y^2 = 14 \quad \text{First equation}$$

$$x^2 - y = 2 \quad \text{Second equation}$$

$$x^2 = y + 2 \quad \text{Second equation solved for } x^2.$$

$$y + 2 + y^2 = 14 \quad \text{Substitute } y + 2 \text{ for } x^2 \text{ in first equation.}$$

$$y^2 + y - 12 = 0 \quad \text{Make one side 0.}$$

$$(y + 4)(y - 3) = 0 \quad \text{Factor.}$$

$$y = -4 \text{ or } y = 3 \quad \text{Solve for } y.$$

Substitute -4 and 3 for y and solve for x^2 .

$$x^2 = y + 2 = -4 + 2 = -2.$$

Since x^2 cannot be negative, $y = -4$ is not a solution.

$$x^2 = y + 2 = 3 + 2 = 5$$

The value of x^2 is 5 .

Chapter 11 Practice Test

1. B

The x -coordinate of the vertex is the average of the x -intercepts. Thus the x -coordinate of the vertex is $x = \frac{-2+6}{2} = 2$. The vertex form of

the parabola can be written as $y = a(x-2)^2 + k$.

Choices A and D are incorrect because the x -coordinate of the vertex is not 2.

Also, the parabola passes through (0,6).

Check choices B and C.

$$\text{B) } y = -\frac{1}{2}(x-2)^2 + 8$$

$$6 = -\frac{1}{2}(0-2)^2 + 8 \quad \text{Correct.}$$

$$\text{C) } y = -\frac{1}{2}(x-2)^2 + 9$$

$$6 = -\frac{1}{2}(0-2)^2 + 9 \quad \text{Not correct.}$$

Choice B is correct.

2. C

$$(x+y)^2 = 324 \Rightarrow x^2 + 2xy + y^2 = 324$$

$$x^2 + y^2 = 324 - 2xy$$

$$(x-y)^2 = 16 \Rightarrow x^2 - 2xy + y^2 = 16$$

$$\Rightarrow x^2 + y^2 = 16 + 2xy$$

Substituting $16 + 2xy$ for $x^2 + y^2$ in the equation

$$x^2 + y^2 = 324 - 2xy \text{ yields}$$

$$16 + 2xy = 324 - 2xy.$$

Solving this equation for xy yields $xy = 77$.

3. A

From the graph we read the length of AD , which is 9. Let the length of $CD = w$.

Perimeter of rectangle $ABCD$ is 38.

$$2 \cdot 9 + 2w = 38 \Rightarrow 2w = 20 \Rightarrow w = 10$$

Therefore, the coordinates of B are $(-1,10)$

and the coordinates of C are $(8,10)$.

The equation of the parabola can be written in vertex form as $y = a(x-3)^2$.

Now substitute 8 for x and 10 for y in the

equation. $10 = a(8-3)^2$. Solving for a gives

$$a = \frac{10}{25} = \frac{2}{5}. \text{ Choice A is correct.}$$

4. A

$$(ax+b)(2x-5) = 12x^2 + kx - 10$$

FOIL the left side of the equation.

$$2ax^2 + (-5a+2b)x - 5b = 12x^2 + kx - 10$$

By the definition of equal polynomials, $2a = 12$, $-5a + 2b = k$, and $5b = 10$. Thus, $a = 6$ and $b = 2$, and $k = -5a + 2b = -5(6) + 2(2) = -26$.

5. D

$$h = -\frac{1}{2}gt^2 + v_0t + h_0$$

In the equation, $g = 9.8$, initial height $h_0 = 40$, and initial speed $v_0 = 35$. Therefore, the equation

$$\text{of the motion is } h = -\frac{1}{2}(9.8)t^2 + 35t + 40.$$

Choice D is correct.

6. C

In the quadratic equation, $y = ax^2 + bx + c$, the x -coordinate of the maximum or minimum point

$$\text{is at } x = -\frac{b}{2a}.$$

Therefore, the object reaches its maximum height

$$\text{when } t = -\frac{35}{2(-4.9)} = \frac{25}{7}.$$

7. A

The object reaches to its maximum height when

$$t = \frac{25}{7}. \text{ So substitute } t = \frac{25}{7} \text{ in the equation.}$$

$$h = -4.9\left(\frac{25}{7}\right)^2 + 35\left(\frac{25}{7}\right) + 40 = 102.5$$

To the nearest meter, the object reaches a maximum height of 103 meters.

8. B

Height of the object is zero when the object hits the ground.

$$0 = -4.9t^2 + 35t + 40$$

Use quadratic formula to solve for t .

$$\begin{aligned} t &= \frac{-35 \pm \sqrt{35^2 - 4(-4.9)(40)}}{2(-4.9)} \\ &= \frac{-35 \pm \sqrt{2009}}{-9.8} \approx \frac{-35 \pm 44.82}{-9.8} \end{aligned}$$

Solving for t gives $t \approx -1$ or $t \approx 8.1$.
Since time cannot be negative, the object hits the ground about 8 seconds after it was thrown.

9. B

When an object hits the ground, $h = 0$.

$h_0 = 150$ is given.

$$0 = -16t^2 + 150$$

Substitution

$$16t^2 = 150$$

Add $16t^2$ to each side.

$$t^2 = \frac{150}{16}$$

Divide each side by 16.

$$t = \sqrt{\frac{150}{16}} \approx 3.06$$