
Exercise - Permutations and Combinations

Questions 1 and 2 refer to the following information.

A hiker is going to hike a mountain where there are four trails to the top of the mountain.

1 _____

In how many different ways can he hike up and down the mountain?

2 _____

If the hiker does not want to take the same trail both ways, in how many different ways can he hike up and down the mountain?

3 _____

In how many ways can the letters of the word SUNDAY be arranged using only 3 of the letters at a time?

Questions 4 and 5 refer to the following information.

Sixteen players participated in a tennis tournament. Three players will be awarded for first, second, and third prize.

4 _____

In how many different ways can the first, second, and third prizes be awarded?

5 _____

How many different groups of 3 people can get prizes?

6 _____

How many different four-letter patterns can be formed from the word MATH if the letters cannot be used more than once?

Exercise - Rules of Probability

Questions 1 and 2 refer to the following information.

A bag contains 15 balls, numbered 1 through 15.

1

What is the probability of selecting a number that is odd or a multiple of 5?

2

A ball is selected at random then replaced in the bag. A second selection is then made. What is the probability that the first number is a prime number and the second number is a multiple of 3?

3

$$S = \{-5, -2, -1, 4\} \quad T = \{-2, 3, 7\}$$

Product $p = s \cdot t$ is formed from the two sets above, in which s is a number from set S and t is a number from set T . What is the probability that the product $s \cdot t$ will be a positive number?

Questions 4 and 5 refer to the following information.

Janis is making a flight reservation for her business trip. The travel agent informs that the probability that her flight to Phoenix will arrive on schedule is 90% and the probability that her flight from Phoenix to Atlanta will arrive on schedule is 80%.

4

What is the probability that both flights arrive on schedule?

5

What is the probability that her flight to Phoenix is on schedule but her flight from Phoenix to Atlanta is not?

6

In a box of 12 headlamps 3 are defective. If you choose two headlamps without replacement, what is the probability that both headlamps are defective?

Exercise - Categorical Data and Conditional Probabilities

Questions 1- 5 refer to the following information.

The table below shows the number of college faculty members in three departments: biological sciences, education, and social sciences.

	Biological Sciences	Education	Social Sciences	Total
Male	10	26	19	55
Female	15	21	17	53
Total	25	47	36	108

1

What is the probability that a randomly chosen faculty member is a female given that she is from Biological Sciences?

2

What is the probability that a randomly chosen faculty member is a male or from Social Sciences?

3

What is the probability that a randomly chosen faculty member is a female from Education department or a male from Social Sciences?

4

What is the probability that a randomly chosen faculty member is from Biological Sciences given that the faculty member is a male?

5

For Biological Science and Education faculties combined, $\frac{1}{6}$ of the female and $\frac{1}{4}$ of the male faculty members are associate professors. If a person is randomly chosen from these two departments, what is the probability that a faculty member is an associate professor?

Answer Key

Section 9-1

1. 16 2. 12 3. 120 4. 3360 5. 560
6. 24

Section 9-2

1. $\frac{3}{5}$ 2. $\frac{2}{15}$ 3. $\frac{5}{12}$ 4. 0.72 5. 0.18
6. $\frac{1}{22}$

Section 9-3

1. $\frac{3}{5}$ 2. $\frac{2}{3}$ 3. $\frac{10}{27}$ 4. $\frac{2}{11}$ 5. $\frac{5}{24}$

Chapter 9 Practice Test

1. C 2. B 3. A 4. D 5. B
6. D 7. C 8. C

Answers and Explanations**Section 9-1**

1. 16

There are 4 different ways of going up and 4 different ways of going down, so there are 4×4 , or 16 different ways he can hike up and down.

2. 12

If the hiker does not want to take the same trail both ways, then there are 4 different ways of going up and 3 different ways of going down, so there are 4×3 , or 12 different ways he can hike up and down.

3. 120

Use ${}_nP_r$, since order is considered.

$${}_6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1}} = 120$$

Therefore, there are 120 ways the 6 letters in SUNDAY can be arranged when the letters are taken 3 at a time.

4. 3360

16	15	14
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The first choice can be any one of the 16 players, the second choice can be any one of the 15 remaining players, and the third choice can be any one of the 14 remaining players. Therefore, there are $16 \times 15 \times 14$, or 3,360 different ways the first, second, and third prizes are awarded.

5. 560

In this case, order does not matter. We must find the combination of 16 people taken 3 at a time.

$${}_{16}C_3 = \frac{16!}{3!(16-3)!} = \frac{16!}{3! \times 13!} = \frac{16 \cdot 15 \cdot 14 \cdot 13!}{3 \cdot 2 \cdot 1 \cdot 13!} = 560$$

6. 24

4	3	2	1
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The first choice can be any one of the 4 letters, the second choice can be any one of the 3 remaining letters, the third choice can be any one of the 2 remaining letters and the fourth choice is the 1 remaining letter. Therefore, there are $4 \times 3 \times 2 \times 1$, or 24 different ways the 4 letters in MATH can be arranged.

Section 9-2

1. $\frac{3}{5}$

There are 8 odd numbers: 1, 3, 5, 7, 9, 11, 13, and 15. There are 3 multiples of 5: 5, 10, and 15. Therefore, there are 9 numbers which are either odd or a multiple of 5.

$$P(\text{odd or a multiple of 5}) = \frac{9}{15} = \frac{3}{5}$$

2. $\frac{2}{15}$

There are 6 prime numbers: 2, 3, 5, 7, 11, and 13.

$$\text{Therefore, } P(\text{prime}) = \frac{6}{15}.$$

There are 5 multiples of three: 3, 6, 9, 12, and 15.

$$\text{Therefore, } P(\text{multiples of 3}) = \frac{5}{15}.$$

The probability that the first number is a prime number and the second number is a multiple of 3

$$\text{is } \frac{6}{15} \times \frac{5}{15}, \text{ or } \frac{2}{15}.$$

3. $\frac{5}{12}$

$$S = \{-5, -2, -1, 4\} \quad T = \{-2, 3, 7\}$$

Make a table of the possible products of $p = s \cdot t$.

$$(-5) \cdot (-2) = 10, (-5) \cdot 3 = -15, (-5) \cdot 7 = -35$$

$$(-2) \cdot (-2) = 4, (-2) \cdot 3 = -6, (-2) \cdot 7 = -14$$

$$(-1) \cdot (-2) = 2, (-1) \cdot 3 = -3, (-1) \cdot 7 = -7$$

$$4 \cdot (-2) = -8, 4 \cdot 3 = 12, 4 \cdot 7 = 28$$

There are 12 products, 5 of which are positive numbers. Therefore,

$$P(\text{product is a positive number}) = \frac{5}{12}$$

4. 0.72

The probability that both flights arrive on schedule is $0.9 \times 0.8 = 0.72$.

5. 0.18

The probability that her flight to Phoenix is on schedule but her flight from Phoenix to Atlanta is not on schedule is $0.9 \times (1 - 0.8) = 0.18$.

6. $\frac{1}{22}$

If your first selection is a defective headlamp, 11 headlamps will be left and 2 of them will be defective. The probability that both headlamps

are defective is $\frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$.

Section 9-3

1. $\frac{3}{5}$

	Biological Sciences	Education	Social Sciences	Total
Male	10	26	19	55
Female	15	21	17	53
Total	25	47	36	108

There are 25 faculty members in Biological Sciences, and 15 of them are female. Therefore, the probability that the person chosen is a female given that she is from Biological Sciences is

$$\frac{15}{25} = \frac{3}{5}.$$

2. $\frac{2}{3}$

There are 55 male faculty members and 36 Social Science faculty members. Since the male faculty in Social Sciences are counted twice you must subtract 19, from the sum of 55 and 36.

Therefore, the probability that a randomly chosen faculty member is a male or from Social

Sciences is $\frac{55}{108} + \frac{36}{108} - \frac{19}{108}$, or $\frac{2}{3}$.

3. $\frac{10}{27}$ or 0.37

There are 21 females from Education Department and 19 males from Social Sciences. Therefore, the probability that a randomly chosen faculty member is a female from Education Department or a male

from the Social Sciences is $\frac{21}{108} + \frac{19}{108}$, or $\frac{10}{27}$.

4. $\frac{2}{11}$

There are 10 male faculty members in Biological Sciences out of 55 males. Therefore, the probability that a randomly chosen faculty member is from the Biological Sciences given that he is a male is

$$\frac{10}{55} = \frac{2}{11}.$$

5. $\frac{5}{24}$

The number of females in both departments combined is $15 + 21$, or 36, and the number of males in both departments combined is $10 + 26$, or 36. The number of female associate professors

in both department combined are $36 \times \frac{1}{6} = 6$ and

the number of male associate professors in both department combined are $36 \times \frac{1}{4} = 9$.

There are $25 + 47$, or 72, faculty members in both departments combined and $6 + 9$, or 15, associate professors in both departments combined. Therefore, if a person is randomly chosen from these two departments, the probability that a faculty member is an associate professor is

$$\frac{15}{72} = \frac{5}{24}.$$

Chapter 9 Practice Test

1. C

	Economics	History	Music
Male	24	20	19
Female	18	22	17

There are 120 student total and 42 students are History majors. Therefore, the probability that the student is a History major is $\frac{42}{120}$.

2. B

There are $24 + 20 + 19 = 63$ male students. If a male student is selected at random, the probability that he is a Music major is $\frac{19}{63} \approx 0.302$.

3. A

The probability that the student is a male Economics major is $\frac{24}{120}$.

4. D

There are $19 + 17$, or 36, Music majors. The probability that a Music major selected at random is a female is $\frac{17}{36} \approx 0.472$.

5. B

	Under 30	30 or older	Total
Male	3		12
Female			20
Total	8	24	32

There are 3 males under age of 30. The number of males 30 years or older is $12 - 3 = 9$. Therefore, the number of females 30 years or older is $24 - 9 = 15$. The probability that the player will be either a male under age 30 or a female aged 30 or older is $\frac{3+15}{32} = \frac{18}{32}$.

6. D

There are 15 females who are aged 30 or older. If a person is selected at random from the 30 or older player group, the probability that the person is a female is $\frac{15}{24}$.

7. C

Number of Visits to Movie Theaters by Students

	None	1 to 2	3 or more
Juniors	x	$2x$	$\frac{1}{2}x$
Seniors	y	$\frac{5}{2}y$	$\frac{1}{2}y$

There are 168 juniors and 152 seniors. Therefore, $x + 2x + \frac{1}{2}x = 168$, and $y + \frac{5}{2}y + \frac{1}{2}y = 152$.

Solving the equations give $x = 48$ and $y = 38$.

There are $2x + \frac{1}{2}x = \frac{5}{2}x = \frac{5}{2}(48) = 120$ juniors

and $\frac{5}{2}y + \frac{1}{2}y = 3y = 3(38) = 114$ seniors who visited movie theaters at least once.

If a student is selected at random from those who visited movie theaters at least once, the probability that the student is a junior is $\frac{120}{120+114}$, or $\frac{20}{39}$.

8. C

Seniors who visited movie theaters 1 or 2 times is $\frac{5}{2}y = \frac{5}{2}(38) = 95$.

The probability that the student is a senior and visited movie theaters 1 or 2 times is

$$\frac{95}{320} \approx 0.297$$

